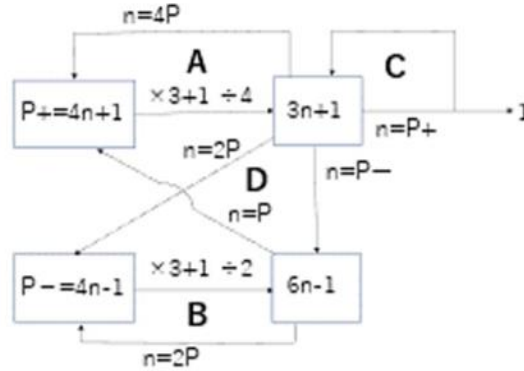


Collatz problem

Hidetomo Tohmori 2021/10/01



Conceptual diagram of Collatz problem

Division by 2 is omitted in above diagram.

Every odd number P is expressed in the form $P_+ = 4n + 1$ or $P_- = 4n - 1$.

In loop **A**, $n = 2^{2l}k$ of $3n + 1$ is converted to $n = 3^l k$. When $n = P_-$, $3n + 1$ is sent to $6n - 1$. When $n = P_+$, $3n + 1$ is sent to loop **C**. When $n = 2^{2l+1}k$, $3n + 1 = P_-$ is sent to P_- . k is odd number.

$$\begin{aligned} n = 2^{2l}k & \quad 3n + 1 = 3 \times 2^{2l}k + 1 \rightarrow 3 \times 3^l k + 1 = 3P + 1 \\ n = 2^{2l+1}k & \quad 3n + 1 = 3 \times 2^{2l+1}k + 1 \rightarrow 2 \times 3^{l+1}k + 1 = P_- \end{aligned}$$

In loop **B**, even number $n = 2^l j$ of $6n - 1$ is converted to odd number $n = 3^l j$. When the n is converted to odd number, $6n - 1$ is converted to P_+ from P_- and is sent to P_+ of loop **A**. j is odd number.

$$n = 2^l j \quad 6n - 1 = 6 \times 2^l j - 1 \rightarrow 6 \times 3^l j - 1 = P_+$$

In loop **C**, when $n = 4m + 1 = P_+$, $3n + 1$ is converted to $3m + 1$. In addition, when $m = P_+$, the above conversion is repeated.

Loop **D** is repeated when n of $3n + 1$ is P_- .

Since the conversion of 2^l to 3^l in loops **A** and **B** is unidirectional, it is considered that there is no infinite cycle.

As described above, any odd number P is output as 1 from loop **C** by cyclic conversion in loops **A**, **B**, **C** and **D**.