A proof of Fermat's Last Theorem ($p = 4$)

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Abstract

If Fermat's equation with an exponent of prime p holds in arithmetic operation, it always holds in modulus operation by factor $(2p+1)$ regardless of whether factor $(2p+1)$ is prime or not.

But, even if Fermat's equation holds in modulus operation by factor $(2p+1)$, it does not necessarily hold in arithmetic operation.

However, Fermat's equation never holds in arithmetic operations unless it holds in modulus operation by factor $(2p+1)$.

Therefore, by proving that Fermat's equation with exponents of numbers 4 does not hold in modulus operations with prime numbers 5, it is proved that Fermat's equations do not hold in arithmetic operations.

As a result, Fermat's Last Theorem ($p = 4$) is proved.

1. Introduction

Fermat's equation with an exponent of number 4 is as follows.

Natural number A, B and C are coprime.

$$A^4 + B^4 = C^4 \qquad (1.1)$$

$\left(\frac{Q}{R}\right)$ is defined as remainder (modulo) when natural number Q is modulo operated by natural number R.

The notation in the center is correct, but it is written as shown on the right side.

$$\left(\frac{QS}{RT}\right) = \left(\frac{\left(\frac{Q}{RT}\right)\left(\frac{S}{RT}\right)}{RT}\right) = \left(\frac{Q}{RT}\right)\left(\frac{S}{RT}\right) \qquad \left(\frac{Q \pm S}{RT}\right) = \left(\frac{\left(\frac{Q}{RT}\right) \pm \left(\frac{S}{RT}\right)}{RT}\right) = \left(\frac{Q}{RT}\right) \pm \left(\frac{S}{RT}\right)$$

2. Fermat's equation with exponent of number

4 does not hold in modulus operations with prime number 5.

2.1 Either of natural numbers A and B contain prime number 5

If none of the natural numbers A, B or C contain prime numbers 5, Fermat's equation does not hold in modulus operations with prime numbers 5.

$$\left(\frac{A^4+B^4}{5}\right) = \left(\frac{A^4}{5}\right) + \left(\frac{B^4}{5}\right) = 1 + 1 = 2 \qquad \left(\frac{C^4}{5}\right) = 1 \qquad \left(\frac{A^4+B^4}{5}\right) \neq \left(\frac{C^4}{5}\right)$$

When the natural number C contains prime numbers 5, Fermat's equation does not hold in modulus operations with prime numbers 5.

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Therefore, either of the natural numbers A or B include prime number 5.

Then, in the following, it is assumed that the natural number A contain prime number 5.

2.2 Fermat's equation is factored

There are always natural numbers U and V that hold the following equations (2.2.1) and (2.2.2).

The natural numbers U, V, A and B are prime to each other.

$$C^2 - B^2 = U \qquad (2.2.1)$$

$$C^2 + B^2 = V \qquad (2.2.2)$$

The following equation (2.2.3) holds.

$$(C^2 - B^2)(C^2 + B^2) = (C^4 - B^4) = A^4 = UV \qquad (2.2.3)$$

Then, since the natural numbers U and V are prime to each other, the following equation holds.

Natural numbers X and Y are prime to each other.

$$U = X^4 \qquad V = Y^4 \qquad XY = A$$

Then, the following equations (2.2.4) and (2.2.5) hold.

$$C^2 - B^2 = U = X^4 \qquad (2.2.4)$$

$$C^2 + B^2 = V = Y^4 \qquad (2.2.5)$$

Since natural number A contain prime number 5, either of natural numbers X or Y includes prime number 5.

However, even if either of the natural numbers X or Y contains prime number 5, any of the above equation (2.2.4) or (2.2.5) does not hold in the modulus operation with prime number 5.

When natural number X contains prime number 5, the equation (2.2.5) does not hold in the modulus operation with prime number 5.

$$\left(\frac{C^2+B^4}{5}\right) = 2\left(\frac{C^2}{5}\right) = \pm 2 \qquad \left(\frac{Y^4}{5}\right) = 1 \qquad \left(\frac{C^2+B^2}{5}\right) \neq \left(\frac{Y^4}{5}\right)$$

When natural number Y contains prime number 5, the equation (2.2.4) does not hold in the modulus operation with prime number 5.

$$\left(\frac{C^2-B^2}{5}\right) = 2\left(\frac{C^2}{5}\right) = \pm 2 \qquad \left(\frac{Y^4}{5}\right) = 1 \qquad \left(\frac{C^2+B^2}{5}\right) \neq \left(\frac{Y^4}{5}\right)$$

Therefore, there are no natural numbers that hold Fermat's equation (1.1) with exponents of number 4.

3. Conclusion

Since Fermat's equation with exponent of number 4 does not hold in modulus operation with prime number 5, Fermat’s equation never holds in arithmetic operation.

Therefore, there are no natural numbers A, B and C that hold Fermat's equation with exponent of number 4.

Thus, Fermat's last theorem ($p = 4$) has been proven.

4. References

[1] “Sophie Germain.” Encyclopaedia Britannica Online. Encyclopaedia Britannica Inc., 2013.
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