A simple generation formula of prime number

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Abstract

A simple generation formula of prime number is derived from the formula for sum of squares of natural numbers. The simple generation formula generates two sequences of prime numbers. Every prime number belongs to one of the two sequences. But the simple generation formula generates non-prime numbers (multiple of prime number). Therefore, method for removing non-prime numbers from the two sequences is examined.

1. Beginning

A simple generation formula of prime number is derived from the formula for sum of squares of natural numbers. The simple generation formula generates two sequences of prime numbers. Every prime number belongs to one of the two sequences. But the simple generation formula generates non-prime numbers (multiple of prime number). Therefore, method for removing non-prime numbers from the two sequences is examined.

2. Derivation of generation formula

The formula for sum of squares of natural numbers is as follows.

$\sum_{i=1}^{k} i^2 = 1 + 2^2 + 3^2 + \cdots + (k-2)^2 + (k-1)^2 + k^2 = k(K+1)(2k+1)/6$

Since the sum of squares of natural number is an integer, $k(K+1)(2k+1)$ is divisible by the number 6.

When $p_k = 2k+1$ is prime number, $(2k+1)$ cannot be divided by the number 6.

Then either of k or $k+1$ is divisible by number 3 because either is even.

As a result, the generation formula of prime number p_k has the following two sequences.

m is natural number

$k = 3m \qquad\qquad p_k = 2k + 1 = 6m + 1$

$k + 1 = 3m \qquad\qquad p_k = 2k + 2 - 1 = 6m - 1$

The two sequences are distinguished and written as follows.

$p_k = 6m + 1 = p_{m+}$

$p_k = 6m - 1 = p_{m-}$

$m =$	1	2	3	4	5	6	7	8	9	10	11	12	13
$p_{m+} = 6m + 1 =$	7	13	19	**25**	31	37	43	49	**55**	61	67	73	79
$p_{m-} = 6m - 1 =$	5	11	17	23	29	**35**	41	47	53	59	**65**	71	**77**

$m =$	14	15	16	17	18	19	20	21	22	23	24	25	26
$p_{m+} = 6m + 1 =$	**85**	91	97	103	109	**115**	**121**	127	133	139	**145**	151	157
$p_{m-} = 6m - 1 =$	83	89	**95**	101	107	113	**119**	**125**	131	137	**143**	149	**155**

$m =$	27	28	29	30	31	32	33	34	35	36	37	25	26
$p_{m+} = 6m + 1 =$	163	**169**	**175**	181	187	193	199	**205**	211	217	223	229	**235**
$p_{m-} = 6m - 1 =$	161	167	173	179	**185**	191	197	203	209	**215**	221	227	233

As described above, every prime number p_k belongs to one of the two sequences generated in the order of m.

Since m is natural number and is infinite, the prime number p_k is generated infinitely.

However, bold nonprime number (multiple of prime number) are also generated.

Therefore, the method for removing the bold non-prime numbers is examined below.

3. Removal of non-prime numbers

The value $m = ij$ corresponding to non-prime number $p_k = p_{m\pm}$ (multiple of prime number $p_{i\pm}$) can be calculated as follows.

n is natural number

$m = ij$

$p_{i+} = 6i + 1$

$$p_{m+} = 6m + 1 = 6ij + 1 = (6i + 1)j - j + 1 = p_{i+}j - (j - 1)$$

$j = np_{i+} + 1$

$m = ij = i(np_{i+} + 1) = nip_{i+} + i$

$$p_{m-} = 6m - 1 = 6ij - 1 = (6i + 1)j - j - 1 = p_{i+}j - (j + 1)$$

$$j = np_{i+} - 1$$

$$m = ij = i(np_{i+} - 1) = nip_{i+} - i$$

$$m = ij$$

$$p_{i-} = 6i - 1$$

$$p_{m+} = 6m + 1 = 6ij + 1 = (6i - 1)j + j + 1 = p_{i-}j + (j + 1)$$

$$j = np_{i-} - 1$$

$$m = ij = i(np_{i-} - 1) = nip_{i-} - i$$

$$p_{m-} = 6m - 1 = 6ij - 1 = (6i - 1)j + j - 1 = p_{i-}j + (j - 1)$$

$$j = np_{i-} + 1$$

$$m = ij = i(np_{i-} + 1) = nip_{i-} + i$$

Example 1.

When non-prime number $p_{m\pm}$ is multiple of prime number 5, the value m is calculated as.

n is natural number

$$p_{m+} = 6m + 1 = 5m + m + 1$$

$$m + 1 = 5n$$

$n =$ 1 2 3 4 5 6 7 ……

$m = 5n - 1 =$ 4 9 14 19 24 29 34 ……

$p_{m+} = 6m + 1 =$ 25 55 85 115 145 175 205 ……

$$p_{m-} = 6m - 1 = 5m + m - 1$$

$$m - 1 = 5n$$

$n =$ 1 2 3 4 5 6 7 ……

$m = 5n + 1 =$ 6 11 16 21 26 31 36 ……

$p_{m-} = 6m - 1 =$ 35 65 95 125 155 185 215 ……

As described above, non-prime number $p_{m\pm}$ (multiple of prime number 5) appears in the value m.

Example 2.

When non-prime number $p_{m\pm}$ is multiple of prime number 7, the value m is calculated as.

n is natural number

$$p_{m+} = 6m + 1 = 7m - m + 1$$

$$m - 1 = 7n$$

$n = \;\; 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \ldots\ldots$

$m = 7n + 1 = \;\; 8 \quad 15 \quad 22 \quad 29 \quad 36 \quad 43 \quad 50 \;\ldots\ldots$

$p_{m+} = 6m + 1 = 49 \quad 91 \quad 133 \;\; 175 \;\; 217 \;\; 259 \;\; 301 \ldots\ldots$

$p_{m-} = 6m - 1 = 7m - m - 1$

$m + 1 = 7n$

$n = \;\; 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \ldots\ldots$

$m = 7n - 1 = \;\; 6 \quad 13 \quad 20 \quad 27 \quad 34 \quad 41 \quad 48 \;\ldots\ldots$

$p_{m-} = 6m - 1 = 35 \quad 77 \quad 119 \;\; 161 \;\; 203 \;\; 245 \;\; 287 \ldots\ldots$

As described above, non-prime number $p_{m\pm}$ (multiple of prime number 5) appears in the value m.

Example 3.

When non-prime number $p_{m\pm}$ is multiple of prime number 11, the value m is calculated as.

n is natural number

$m = ij = 2j \qquad i = 2$

$p_{m+} = 6m + 1 = 12j + 1 = 11j + j + 1$

$j + 1 = 11n \qquad j = 11n - 1$

$n = \;\; 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \;\ldots\ldots$

$m = 2j = 2(11n - 1) = \;\; 20 \quad 42 \quad 64 \quad 86 \quad 108 \quad 130 \ldots\ldots$

$p_{m+} = 6m + 1 = 121 \;\; 253 \;\; 385 \;\; 517 \;\; 649 \;\; 781 \ldots\ldots$

$p_{m-} = 6m - 1 = 12j - 1 = 11j + j - 1$

$j - 1 = 11n \qquad j = 11n + 1$

$n = \;\; 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \;\ldots\ldots$

$m = 2j = 2(11n + 1) = \;\; 24 \quad 46 \quad 68 \quad 90 \quad 112 \quad 134 \ldots\ldots$

$p_{m-} = 6m - 1 = 143 \;\; 275 \;\; 407 \;\; 539 \;\; 671 \;\; 803 \ldots\ldots$

As described above, non-prime number $p_{m\pm}$ (multiple of prime number 11) appears in the value m.

Example 4.

When non-prime number $p_{m\pm}$ is multiple of prime number 13, the value m is calculated as.

n is natural number

$m = 2m'$

$p_{m+} = 6m + 1 = 12m' + 1 = 13m' - m' + 1$

$m' - 1 = 13n$

$m' = 13n + 1$

$n =$	1	2	3	4	5	6	7	……
$m = 2m' = 2(13n+1) =$	28	54	80	106	132	158	184	……
$p_{m+} = 6m + 1 =$	157	325	481	637	793	949	1105	……

$p_{m-} = 6m - 1 = 12m' - 1 = 13m' - m' - 1$

$m' + 1 = 13n$

$m' = 13n - 1$

$n =$	1	2	3	4	5	6	7	
$m = 2m' = 2(13n-1) =$	24	50	76	102	128	154	180	……
$p_{m-} = 6m - 1 =$	143	299	455	611	767	923	1079	……

Example 5.

When non-prime number $p_{m\pm}$ is multiple of prime number 17, the value m is calculated as.

n is natural number

$m = 3m'$

$p_{m+} = 6m + 1 = 18m' + 1 = 17m' + m' + 1$

$m' + 1 = 17n$

$m' = 17n - 1$

$n =$	1	2	3	4	5	6	7	……
$m = 3m' = 3(17n-1) =$	48	99	150	201	252	303	354	……
$p_{m+} = 6m + 1 =$	289	595	901	1201	1513	1819	2125	……

$p_{m-} = 6m - 1 = 18m' - 1 = 17m' + m' - 1$

$m' - 1 = 17n$

$m' = 17n + 1$

$n =$	1	2	3	4	5	6	7	……
$m = 3m' = 3(17n+1) =$	54	105	156	207	258	309	360	……
$p_{m-} = 6m - 1 =$	323	629	935	1241	1547	1853	2159	……

Example 6.

When non-prime number $p_{m\pm}$ is multiple of prime number 19, the value m is calculated as.

n is a positive integer

$m = ij = 3j \qquad i = 3$

$p_{m+} = 6m + 1 = 18j + 1 = 19j - j + 1$

$j - 1 = 19n \qquad j = 19n + 1$

$$n = \quad 1 \quad 2 \quad 3 \quad 4 \quad \dots\dots$$

$$m = 3j = 3(19n + 1) = \quad 60 \quad 117 \quad 174 \quad 231 \dots\dots$$

$$p_{m+} = 6m + 1 = 361 \quad 703 \quad 1045 \quad 1387 \quad \dots\dots$$

$$p_{m-} = 6m - 1 = 18j - 1 = 19j - j - 1$$

$$j + 1 = 19n \qquad j = 19n - 1$$

$$n = \quad 1 \quad 2 \quad 3 \quad \dots\dots$$

$$m = 3j = 3(19n - 1) = \quad 54 \quad 111 \quad 168 \dots\dots$$

$$p_{m-} = 6m - 1 = 323 \quad 665 \quad 1007 \dots\dots$$

Example 7.

When non-prime number $p_{m\pm}$ is multiple of prime number 23, the value m is calculated as.

n is natural number

$$m = 4m'$$

$$p_{m+} = 6m + 1 = 24m' + 1 = 23m' + m' + 1$$

$$m' + 1 = 23n$$

$$m' = 23n - 1$$

$$n = \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \dots\dots$$

$$m = 4m' = 4(23n - 1) = \quad 88 \quad 180 \quad 272 \quad 364 \quad 456 \quad 548 \quad 640 \dots\dots$$

$$p_{m+} = 6m + 1 = 529 \quad 1081 \quad 1633 \quad 2185 \quad 2737 \quad 3289 \quad 3841 \dots\dots$$

$$p_{m-} = 6m - 1 = 24m' - 1 = 23m' + m' - 1$$

$$m' - 1 = 23n$$

$$m' = 23n + 1$$

$$n = \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \dots\dots$$

$$m = 4m' = 4(23n + 1) = \quad 96 \quad 188 \quad 280 \quad 372 \quad 464 \quad 556 \quad 648 \dots\dots$$

$$p_{m-} = 6m - 1 = 575 \quad 1127 \quad 1679 \quad 2231 \quad 2783 \quad 3335 \quad 3887 \dots\dots$$

4. Conclusion

Every prime number is generated by removing non-prime number (multiple of prime number) from the two sequences.

On the contrary, in order to determine whether or not a given integer is prime number, firstly, it is determined which of the two sequences the given integer corresponds to, and secondly, it is judged whether or not the value m of the given integer matches the value m of non-prime number (multiple of prime number).

When the given integer to be judged becomes huge, the number of non-prime numbers to judge becomes huge.

However, the calculation of the value m corresponding to each non-prime number is only addition and subtraction and multiplication, so it may be easy in devising computer software.

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