Apsidal precession

It is assumed that an arbitrary position in the gravitational field belongs to the inertia system which has the time axis t' with the light speed C and the radial axis R' with the relative speed $U(R)$. The squared light speed C'^2 in the gravitational field could be obtained from the coordinate transformation to the inertia system from the static system.

$$(dS)^2 = (iCdt)^2 = (dS')^2 = (iCdt')^2 + (U(R)dt')^2$$

dS : World space distance

d : Extremely infinitesimal change

i : Imaginary ($i^2 = -1$)

And also, since the light advance distance is invariant, the following formula is formed.

$$Cdt = C'dt'$$

Then, the following formula (1) is obtained.

(1) $C'^2 = C^2 - U^2 \quad U : U(R)$

V : Speed of the mass "m" in the static system

V' : Speed of the mass "m" in the inertia system

$$(dS)^2 = (iCdt)^2 = (dS')^2 = (iCdt')^2 + (V'dt')^2 = (-C^2 + V'^2)dt'^2$$

$$(dS')^2 = (iCdt')^2 = (dS")^2 = (iCdt")^2 + (Udt")^2 = (-C^2 + U^2)dt"^2$$

$$(-C^2 + V^2)dt'^2 = (iCdt)^2$$

$$(-C^2 + U^2)dt"^2 = (iCdt')^2$$

$$(-C^2 + U^2)(-C^2 + V^2)dt"^2 = (-C^2 + V^2)(dt')^2(iC)^2 = (iCdt)^2 (iC)^2$$

$$(C^4C^2 - C^2(V^2 + U^2) + U^2V^2)dt"^2 = -C^4dt^2$$

When V is small enough compared to C, i.e. $0 < V/C \ll 1$, the following formulas are formed.

$$(1 - (V^2 + U^2)/ C^2)dt"^2 = (1 - V'^2/ C^2)dt"^2$$

(2) $V'^2 = V^2 + U^2$

$$mV'^2/2 = mV^2/2 + mU^2/2, \quad d(mV^2/2)/dR = 0$$

$$d(mV'^2/2)/dR = d(mU^2/2)/dR = F = mg \, (g = GM/R^2)$$

$$dU^2/dR = 2GM/R^2$$

(3) $U^2 = 2GM/R$

In Fig.1, the mass m with the speed V vertically passes at the distance R from the gravitational field center of mass M, and then, the advance direction of the mass is rotated by the angle ω for the rotation angle θ of the distance R' from the center.

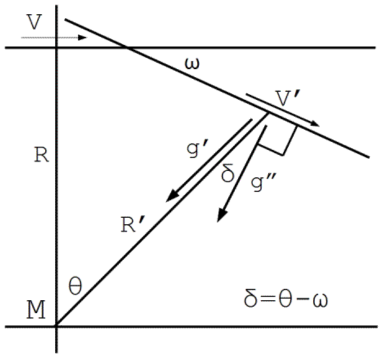

Fig.1

Then, the following formula (4) is formed.

(4) $d\omega/d\theta = 2GM/(V'^2R')$

On the other hand,

v: The speed of the mass at the infinite distance

$V^2 = U^2 + v^2 \qquad U^2 = 2GM/R$

$V'^2 = U'^2 + v^2 \qquad U'^2 = 2GM/R'$

$V'^2 = V^2 + U'^2 - U^2$

$= V^2(1 + 2GM(1/V^2R' - 1/V^2R))$

$R = R'\cos\delta, \quad \delta = \theta - \omega$

$V'^2 = V^2(1 + 2GM((\cos\delta/V^2R) - (1/V^2R))$

$2GM/V^2Rv = 1 \qquad R = \gamma Rv$

$= V^2(1 + (\cos\delta - 1)/\gamma)$

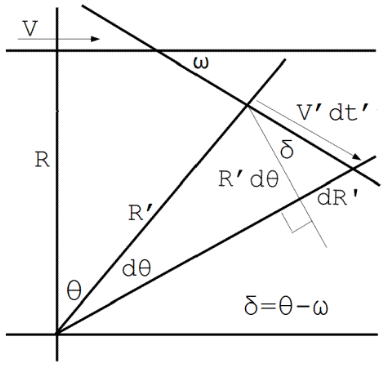

Fig.2

The above formula (4) is rewritten as the following.

(5) $d\delta/d\theta = (\gamma - 1)/(\gamma + \cos\delta - 1)$

$2GM/V^2Rv = 1 \qquad R = \gamma Rv$

When $\gamma > 1$, $d\delta/d\theta > 0$ and then the orbit is parabolic.
When $\gamma = 1$, the orbit is circular.
When $0 < 1 - \gamma \ll 1$, the following formulas are formed.

$R' = R\cos\delta', \quad \delta' = \omega - \theta$

$V'^2 = V^2(1 + 2GM((1/V^2R\cos\delta) - (1/V^2R))$

$2GM/V^2Rv = 1 \qquad R = \gamma Rv$

$V'^2 = V^2(1 + 1/\gamma\cos\delta - 1/\gamma)$

$d\omega/d\theta = 2GM/(V'^2R') = 2GM/(V^2(1 + 1/\gamma\cos\delta - 1/\gamma)\gamma Rv\cos\delta)$

$= 1/((\gamma - 1)\cos\delta + 1)$

(6) $d\delta/d\theta = 1 - d\omega/d\theta = (\gamma - 1)\cos\delta/((\gamma - 1)\cos\delta + 1)$

The apsidal precession of the elliptic orbit is calculated as the following.

$d\delta/d\theta \cong (\gamma - 1)$

$\delta_{2\pi}$: Apsidal precession by one around ($\theta = 0 \sim 2\pi$)

$$\delta_{2\pi} = \int_0^{2\pi}(\gamma - 1)d\theta = 2\pi(\gamma - 1) < 0$$