Special relativity

Special relativity is a coordinate transformation of two inertial systems that move relative to each other at a constant velocity.

For the sake of clarity, one of the above two inertial systems is named the stationary system and the other is named the inertial system.

More specifically, the physical quantity of the inertial system (4D space) observed in the stationary system (4D space) is the coordinate conversion of the physical quantity of the stationary system into the inertial system.

The world distance(interval) dS^2 is preserved (invariant) in the above coordinate transformation in space-time (4D space), just as the distance is preserved (invariant) in the coordinate transformation in 3D space.

The inertial system is moving at a velocity U relative to the stationary system. It is assumed that an object with mass m is stationary in a stationary system. At that time, only time dt elapses in the stationary system.

In the inertial system, the time dt' only elapses, during which the object m moves by the distance Udt'.

C : light speed in the stationary system and coefficient of the time axis

C' : light speed of the inertial system

U : relative speed between the stationary system and the inertial system

dt : Elapsed time in the stationary system

dt': Elapsed time in the inertial system

m : Mass of the object

$\beta^2 = U^2/C^2$

Since the world distances(interval) dS^2 in both systems are equal, the following equation (1) holds.

$$-dS^2 = C^2dt^2 = C^2dt'^2 - U^2dt'^2$$

$$dt^2/dt'^2 = 1 - \beta^2$$

(1) $$dt = dt'\sqrt{1 - \beta^2}$$

Since the propagation distance of light is the same in both systems (This is the principle of invariant speed of light.), the following equation (2) holds.

$$C'^2 dt'^2 = C^2 dt^2$$

$$C'^2 = C^2 dt^2/dt'^2 = C^2(1-\beta^2)$$

(2) $$C' = C\sqrt{1-\beta^2}$$

Here, it is assumed that the object m is moving at the velocity v in the stationary system.

Since the distance that the object m moves is the same in both systems, the following equation (3) holds. It is assumed that the object m is moving at the velocity v' in the inertial system.

$$v'^2 dt'^2 = v^2 dt^2$$

$$v'^2 = v^2(1-\beta^2)$$

(3) $$v' = v\sqrt{1-\beta^2}$$

When the object m moving at the velocity v in the stationary system is accelerated by the force F, the object m is accelerated by the same force F in the inertial system. At that time, the following equation holds.

$$v = v'/\sqrt{1-\beta^2} \qquad dt = dt'\sqrt{1-\beta^2}$$

$$F = m(dv/dt) = m(dv'/dt')/(1-\beta^2) = m'(dv'/dt')$$

The following equation (4) holds.

(4) $$m/(1-\beta^2) = m'$$

Therefore, when the object m moves at the velocity v in the stationary system, the object with the mass m' appears to move at the velocity v' in the inertial system.

The energy of mass M is MC^2 (See **Supplement 4**).

Since the amount of energy is invariant regardless of whether it is the stationary system or the inertial system, the same equation as equation (4) holds.

$$M'C'^2 = MC^2$$

$$M' = MC^2/C'^2 = M/(1-\beta^2)$$

The frequency of light emitted from the object moving at the velocity U with respect to the observation system decreases. In this case, the observation system is the inertial system, and the moving object is the stationary system. This is because the object emits light.

The following equation holds.

ν : Frequency of light in the stationary system (the moving object)

ν' : Frequency of light in the inertial system (the observation system)

Since the total frequencies of light in both systems are equal, the same equation as equation (3) above holds.

$$\nu dt = \nu' dt' \quad dt = dt'\sqrt{1-\beta^2}$$
$$\nu dt'\sqrt{1-\beta^2} = \nu' dt'$$
$$\nu' = \nu\sqrt{1-\beta^2}$$

This means that the wavelength of light is observed to be longer. It is called the redshift of light.

The Newtonian Doppler effect is irrelevant.

Copernican Relativity

The Schwarzschild solution of Einstein's gravitational field equation expresses the world distance (interval) dS^2 of the local space (local inertial system) of the spherically symmetric gravitational field by spherical coordinates.

However, the local space (local inertial system) is not a perfect inertial system. Therefore, the world distance dS^2 contains the non-linear distortion of the radial axis R.

In that local space (local inertial system), the time dt' changes continuously in the radial axis R direction, so the world distance dS^2 must include the nonlinear distortion of the radial axis R.

However, since the world distance dS^2 contains the non-linear distortion of the radial axis R, it is too difficult to describe the motion of light or object in the gravitational field.

In a local space (local inertial system), light propagates by changing its propagation direction without changing its speed C'. However, since the gravitational field is a continuous connection of local spaces (local inertial systems), light propagates in the gravitational field while changing its velocity C' and its propagation direction.

Therefore, the time dt' and the speed of light C' in the local space (local inertial system) are examined.

The trajectory of light in a spherically symmetric gravitational field is drawn on one plane including the center of the gravitational field.

The local space (local inertial system) is represented by a three-dimensional space having time axis t' with coefficient C (speed of light), radial axis R, and rotation angle θ. Similarly, the stationary system (non-gravitational field) is represented by a three-dimensional space having time axis t with coefficient C, radial axis R, and rotation angle θ.

Since the local space (local inertial system) is the inertial system, it is moving at the velocity $U(R)$ with respect to the stationary system (non-gravitational field).

C' : Speed of light in the local inertial system

C : Speed of light in the stationary system (non-gravitational field)

dt' : Elapsed time in the local inertial system

dt : Elapsed time in the stationary system (non-gravitational field)

i : Imaginary number $i^2 = -1$

dS^2 : World distance in the stationary system where only time dt elapses

dS'^2 : World distance in the local inertial system that moves by the distance $U(R)dt'$ as time dt' elapses.

$dS^2 = (iCdt)^2$

$dS'^2 = (iCdt')^2 + (U(R)dt')^2$

$dS^2 = dS'^2$

$(iCdt)^2 = (iCdt')^2 + (U(R)dt')^2$

Since the distance that light propagates is the same in both systems, the following equation holds.

$C'dt' = Cdt$

The above equation derives the following equation (4).

(4) $C'^2 = C^2 - U^2$, $U = U(R)$

When the object $m\,(\ll M)$ invades the gravitational field of the point mass M from infinity at velocity v, the following equation (5) holds (see **Supplement 1** for the derivation of equation (5)).

(5) $U^2 = 2GM/R$

Using equation (5), equation (4) is rewritten as follows.

(6) $C'^2 = C^2 - 2GM/R$

When $C'^2 = 0$, $C^2 - 2GM/R = 0$

$Rs\,(= 2GM/C^2)$: Schwarzschild radius

$\delta = \theta - \omega > 0$

First, the propagation of light is orthogonal to the points at the distance R from the point mass M as shown in

Fig. 1.
After that, as shown in Fig. 1, the light reaches to the distance R' from the point mass M.

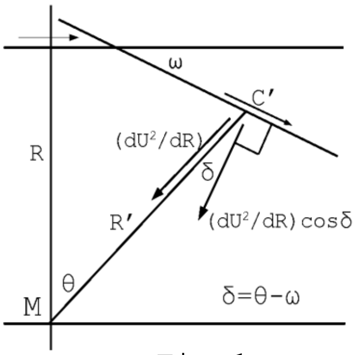


Fig.1

The light rotates around the point mass M by the angle θ.
At that time, the light propagation direction is rotated by the angle ω.

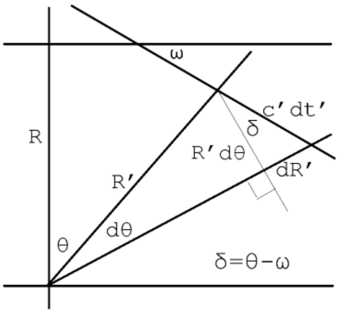


Fig.2

It is assumed that the angular velocity $C'(d\omega/dt')$ in the direction of light propagation is equal to the gravitational acceleration $dU^2 cos\delta/dR$.
Then, the following equation (7) holds.

(7) $$C'(d\omega/dt') = (dU^2/dR)cos\delta$$
$$= C'(d\omega/d\theta)(d\theta/dt')$$

The following equation holds as shown in Fig. 2.

$$d\theta/dt' = C'cos\delta/R'$$

Using the above equation, equation (7) can be rewritten as follows.

$$C'(d\omega/dt') = C'(d\omega/d\theta)(C'cos\delta/R') = (dU^2/dR)cos\delta = 2GMcos\delta/R'^2$$

From the above equation, the following equation (8) is obtained.

(8) $d\omega/d\theta = 2GM/C'^2R'$

Using equation (6), equation (8) is rewritten as the following equation (9).

$$d\omega/d\theta = (2GM/C^2R')/(1 - 2GM/C^2R')$$

$Rs = 2GM/C^2$: Schwarzschild radius

(9) $d\omega/d\theta = (Rs/R')/(1 - Rs/R')$

$R' = R/cos\delta \qquad 2Rs < R$

$R' = Rcos\delta \qquad Rs < R < 2Rs$

See **Supplement 2** for the derivation of the above equation

Schwarzschild solution and Copernican relativity

Here, the Schwarzschild solution in the local space (local inertial system) in a spherically symmetric gravitational field centered on the point mass M is examined.

dt' : Elapsed time in the local inertial system (gravitational field)

dt : Elapsed time in the stationary system (non-gravitational field)

R : Distance from point mass M when the light propagation direction is orthogonal to the radial axis

R' : Distance of the light from point mass M

C : Speed of light in the stationary system and coefficient of time axis and

C' : Speed of light in the inertial system

θ : Angle of rotation of light around point mass M

ω : The angle of rotation in light propagation direction when the angle of rotation of light is θ

d : Small change, that is, differentiation

G : Gravitational constant

$Rs(= 2GM/C^2)$: Schwarzschild radius

The Schwarzschild solution of Einstein's gravitational field equation is written as follows.

$$-dS^2 = C^2(1 - Rs/R')dt'^2 - dR'^2/(1 - Rs/R') - R'^2(d\theta^2 + sin^2\theta d\Phi^2)$$

The above Schwarzschild solution can be rewritten as the following equation (10).

$$C'^2 = C^2(1 - Rs/R') \quad d\Phi = 0$$

$$1/(1 - Rs/R') = 1 + (Rs/R')/(1 - Rs/R')$$

(10) $-dS^2 + (Rs/R')dR'^2/(1 - Rs/R') = C'^2dt'^2 - dR'^2 - R'^2d\theta^2$

The right-hand side of equation (10) is equal to 0 as shown in FIG. 2.

(9) $d\omega/d\theta = (Rs/R')/(1 - Rs/R')$

Using the above equation (9) in the previous section, the above equation (10) is rewritten as in the following equation

(11).

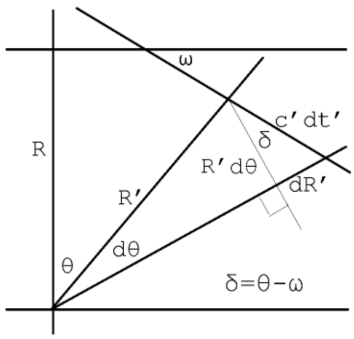


Fig. 2

(11) $-dS^2 + (d\omega/d\theta)dR'^2 = C'^2dt'^2 - dR'^2 - R'^2d\theta^2 = 0$

The above equation (11) shows that the physical quantity $(d\omega/d\theta)dR'^2$ corresponds to the distortion of the radial axis R' which is included in the world distance dS^2 of the Schwarzschild solution.

The right-hand side of equation (11) does not include the nonlinear distortion of the radial axis R'.

Then, the non-linear quantity $(d\omega/d\theta)dR'^2$ cancels the distortion of the radial axis R' included in the world distance dS^2.

Now, it is considered why this happened.

$d\omega/d\theta$ is the ratio of the minute rotation angle $d\theta$ of light when light propagates the distance $C'dt'$ to the minute rotation angle $d\omega$ in light propagation direction.

Due to the radial acceleration $dU^2/dR'(= -2GM/R'^2)$ in the local space (local inertial system), the light cannot propagate straight and the light propagation direction is accelerated in the radial axis R' direction.

Acceleration in the radial axis means changing direction without changing the magnitude of light speed.

As described above, while the light rotates by the minute rotation angle $d\theta$, the light propagation direction rotates by the minute rotation angle $d\omega$.

In the Schwarzschild solution, the radial axis R' is distorted so as to compensate for the bending $(d\omega/d\theta)dR'^2$ of light in the gravitational field.

If it is assumed that light propagates in the local space (local inertial system) while changing its speed C' and the propagation direction, the radial axis R' of the local space (local inertial system) does not need to be distorted.

However, if it is assumed that light travels straight in the local space (local inertial system) at the speed of light C as Schwarzschild solution, the radial axis R' of the local space (local inertial system) must be distorted.

Therefore, in the Schwarzschild solution, the light appears to bend and propagate due to the distortion of the radial axis R'.

But, there is no way to be sure which is true.

However, assuming that light propagates in a local space (local inertial system) while changing its speed of light C' and its propagation direction, it is not necessary to distort the radial axis R'.

Therefore, compared to Schwarzschild solution, the propagation of light and the motion of the object can be described in a visually easy-to-understand manner.

This is Copernican Revolution of relativity.

In Schwarzschild solution, the energy of the gravitational field is understood to be included in the distortion of the radial axis R'.

When the radial axis R' is not distorted, the energy of the gravitational field (see Supplement 6) is considered to be contained in the radial acceleration space created by the gravitational field.

Therefore, the distortion energy of the radial axis R' and the energy of the acceleration space are considered to be equal.

The propagation of distortion on the radial axis R' (gravitational wave) corresponds to the propagation of changes in acceleration (acceleration wave).

Black Hole

Black Hole BH_M determined by $R = \gamma Rs\,(1 < \gamma < 2)$ is considered.

See **Supplement 5** for the derivation of the following equation.

$$R' = Rcos\delta$$

Using the above equation, equation (9) can be rewritten as follows.

$$d\omega/d\theta = (Rs\,/Rcos\delta)/(1 - Rs\,/Rcos\delta)$$

When $R = \gamma Rs\,(1 < \gamma < 2)$, the above equation can be rewritten as follows.

(12) $d\omega/d\theta = 1/(\gamma cos\delta - 1) > 1$

The above equation (12) shows that the trajectory of light is spiral. In other words, light that has entered a gravitational field within twice the Schwarzschild radius Rs cannot escape the gravitational field.

Therefore, when the mass radius R_M of the mass M (see **Supplement 7**) is larger than the Schwarzschild radius Rs and less than twice that, the mass M is Black Hole BH_M.

(13) $Rs < R_M < 2Rs$

There is no celestial substance M whose mass radius R_M is smaller than the Schwarzschild radius Rs.

When the mass radius R_M is smaller than the Schwarzschild radius Rs, a space-time is generated in which no physical quantity (space-time, light or matter, electric charge, etc.) can exist. However, such a situation is physically impossible.

Therefore, when the mass radius R_M tries to be smaller than the Schwarzschild radius Rs, it is considered that the Black Hole BH_M explodes, the mass M disperses, and the Black Hole BH_M disappears.

The apparent mass M_B of the Black Hole BH_M is calculated as follows.

(14) $M\,/(1 - Rs/R_M) = M_B \qquad Rs < R_M < 2Rs$

As is clear from the above equation (14), as the mass radius R_M approaches the Schwarzschild radius Rs, the apparent mass M_B of the Black Hole BH_M increases infinitely.

The apparent mass M_B of the Black Hole BH_M is thought to enable the existence of the supermassive Black Hole BH_{SM}. Such a supermassive Black Hole BH_{SM} is thought to exist in the center of every galaxy.

However, as mentioned above, the supermassive Black Hole BH_{SM} is destined to explode and disappear, so the existence of such a supermassive Black Hole BH_{SM} can be said to be the last appearance of the galaxy.

Supplement 1

When the object $m\,(\ll M)$ invades the gravitational field of point mass M from infinity at velocity v, the following equation holds. It is assumed that the object m is in the local inertial system (gravitational field).

R: Distance of object m from point mass M

$g\,(= GM/R^2)$: Gravitational acceleration acting on the object m

v : Velocity of object m at infinity

V' : Velocity of object m in the local inertial system (gravitational field)

$F(= mg)$: Force acting on the object m in the local inertial system (gravitational field)

$$(iCdt')^2 + (Udt')^2 + (vdt')^2 = (dS')^2 = (iCdt')^2 + (V'dt')^2$$

$$(Udt')^2 + (vdt')^2 = (V'dt')^2$$

$$V'^2 = v^2 + U^2$$

$$mV'^2/2 = mv^2/2 + mU^2/2$$

$$d(mv^2/2)/dR = 0$$

When the energy E is differentiated by dR, the force F acting on the mass M is obtained. In other words, $dE = FdR$. Since $E = mV'^2/2$, the following equation holds.

$$d(mV'^2/2)/dR = d(mU^2/2)/dR = -F = -mg = -mGM/R^2$$

$$dU^2/dR = -2GM/R^2$$

(5) $$U^2 = 2GM/R$$

Supplement 2

With reference to Fig. 2, when $\theta - \omega > 0$, R' is obtained as follows.

$$\delta = \theta - \omega > 0$$

$dR' = C'dt'sin\delta$

$d\theta/dt' = C'cos\delta/R'$

$dR'/dt' = (dR'/d\theta)(d\theta/dt') = C'sin\delta$

$(dR'/d\theta)(C'cos\delta/R') = C'sin\delta$

$d(logR')/d\theta = sin\delta/cos\delta = -d(log(cos\delta))/d\delta$

$logR' = -log(cos\delta) + log\ A$

A: Integral constangt

$R'cos\delta = R'cos(\theta - \omega) = A$

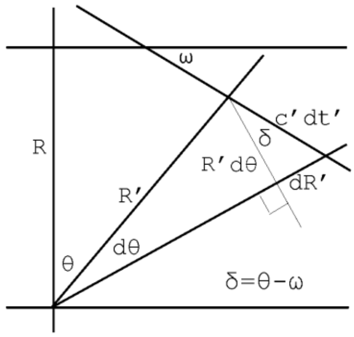


Fig. 2

When the light is orthogonal to the radial axis R,

$\theta = \omega = 0$ $\quad R' = R$

Therefore, $R = A$

Then $\quad R' = R/cos\delta$

Supplement 3

Muon μ life and mileage

Very high-energy primary cosmic rays collide with atoms in the atmosphere to create high-energy (speed v) muons μ. It is known that this muon μ reaches the ground from a high altitude of about $6km$.

When measured on the ground, the stationary muon μ decays into an electron e^- and a neutral micron (Nutrino, γ_μ) with an average life expectancy of $\tau = 2.15 \times 10^{-6}\text{sec}$.

No particle can exceed the speed of light $C = 3.0 \times 10^8 m/s$

Even if this muon μ travels at the speed of light C, its mileage is at most $\tau \times C = 2.15 \times 10^{-6} sec \times 3.0 \times 10^8 m/s = 645m$.

Muon μ cannot reach the ground by traveling the distance $6km$ from high altitude

Therefore, in the following, it is relativistically verified that the muon μ can reach the ground.

Since the average lifetime $\tau = 2.15 \times 10^{-6}\text{sec}$ of the muon μ is in the stationary system, the muon μ is used as the stationary system.

In the stationary system, the muon μ decays after the lapse of time $dt = \tau$. The world distance dS^2 is as follows.

$$dS^2 = C^2 dt^2 = C^2 \tau^2$$

The ground to be observed is an inertial system. In the inertial system, the muon μ decays after the μ particle travels at the velocity v for $6km$ during the time dt'. The world distance dS'^2 is as follows.

$$dS'^2 = C^2 dt'^2 - (6 \times 10^3)^2$$

Since both world distances are equal, the following equation holds.

$$C^2 (2.15 \times 10^{-6})^2 = C^2 dt'^2 - (6 \times 10^3)^2$$

$$C^2 = (3.0 \times 10^8)^2$$

$$C^2 (2.15 \times 10^{-6})^2 + (6 \times 10^3)^2 = C^2 dt'^2$$

$$dt'^2 = (2.15 \times 10^{-6})^2 + (6 \times 10^3)^2/(3.0 \times 10^8)^2 \cong 4.6891 \times 10^{-10}$$

$dt' \cong 2.165 \times 10^{-5} s$

$v = 6 \times 10^3 / dt' \cong 2.771 \times 10^8 m/s < C = 3.0 \times 10^8$

The muon μ reaches the ground from a high altitude of about $6km$ at the speed $\cong 2.771 \times 10^8 m/s$ to collapse which is close to $C = 3.0 \times 10^8$.

This is by no means impossible.

It is shown that time elapses faster in the inertial system than in the stationary system.

Supplement 4

Energy of point mass M

The acceleration of the gravitational field of the point mass M is U^2 differentiated by dR (dU^2/dR) as follows.

$U^2 = 2GM/R$

$dU^2/dR = (d(2GM/R)/dR = -2GM/R^2$

The above acceleration acts on the mass dM at a distance R from the point mass M.

The force F acting on the mass dM is as follows.

$F = 2GMdM/R^2$

When the force F acts on the mass dM over the distance dR, the energy dE is as follows.

$dE = FdR$

The following energy E_{dM} is required to carry the mass dM from the Schwarzschild radius R_s to infinity.

$R_s = 2GM/C^2$

$$E_{dM} = \int dE = \int_{R_s}^{\infty} FdR = \int_{R_s}^{\infty}(2GMdM/R^2)dR = 2GMdM/R_s = C^2dM$$

As described above, the energy E_{dM} is the energy required to carry the point mass dM from the Schwarzschild radius R_s to infinity.

Then, the following equation holds.

$$E_M = \int E_{dM} = \int_0^M C^2dM = MC^2$$

E_M : Energy required to carry point mass M from Schwarzschild radius R_s to infinity

Therefore, $E_M = MC^2$ is the energy of the point mass M itself.

Supplement 5

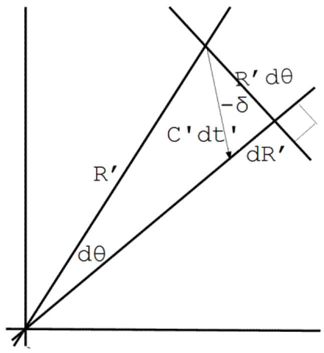


Fig.3

$R' = Rcos\delta$ holds in the case of $R = \gamma Rs\ (1 < \gamma < 2)$.

$$-\delta = \theta - \omega$$

$$dR'/R'd\theta = -tan\delta = -sin\delta/cos\delta$$

$$(dR'/d\theta)(cos\delta/R') = -sin\delta$$

$$d(logR')/d\theta = -sin\delta/cos\delta = d(log(cos\delta))/d\delta$$

$$logR' = log(cos\delta) + logA$$

$logA$: Integral constant

$$R' = Acos\delta,\quad \theta = \omega = 0,\ R' = R = A$$

$$R' = Rcos\delta = \gamma Rs(cos\delta)$$

Using the above equation, equation (9) can be rewritten as follows.

(9) $d\omega/d\theta = (Rs/R')/(1 - (Rs/R'))$

$$d\omega/d\theta = (Rs/Rcos\delta)/(1 - (Rs/Rcos\delta))$$

$$= (1/\gamma cos\delta)/(1 - 1/\gamma cos\delta)$$

$$= 1/(\gamma cos\delta - 1) > 0$$

$$d\delta/d\theta = d\omega/d\theta - 1 = (2 - \gamma cos\delta)/(\gamma cos\delta - 1)$$

$$(2 - \gamma cos\delta) > 0$$

$$= (2 - \gamma cos\delta)(d\omega/d\theta) > 0$$

$$d\omega/d\theta - 1 > 0$$

(13) $d\omega/d\theta > 1$

Supplement 6

The propagation direction of the light propagating around the sun M is bent as follows as shown in Fig. 5.

R : Mass radius of the sun

M : Mass of the sun

$R' = R/cos\delta$: See **Supplement 2** for the derivation of this equation.

ω_p: The angle at which the light propagating around the sun M is bent

$Rs(=2GM/C^2)$: Schwarzschild radius of Sun M

$$Rs \ll R \qquad \omega \ll \theta \qquad \delta = \theta - \omega \cong \theta$$

$$d\omega/d\theta = \mathrm{Rs}(\cos\delta/\mathrm{R})/(1-\mathrm{Rs}(\cos\delta/\mathrm{R})) \cong (Rs/R)cos\theta$$

$$\omega_p = \int d\omega \cong \int_{-\pi/2}^{\pi/2} (Rs/R)cos\theta\, d\theta$$

$$= (Rs/R)(sin\,(\pi/2) - sin\,(-\pi/2))$$

$$\omega_p = 2Rs/R = 4GM/C^2R$$

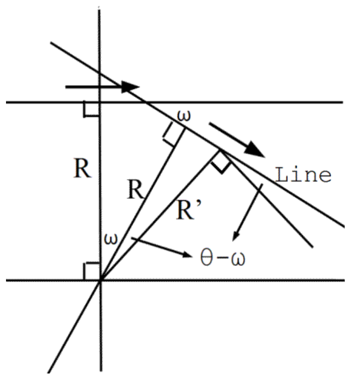


Fig.5

Supplement 7

Black hole mass M and mass radius R_M

If the radial density of mass M is ρ/r^2, R_S becomes indefinite.

If the radial density of the mass M is ρ/r^3, the mass radius R_M is smaller than the Schwarzschild radius R_S.

Assuming that the radial density ρ of the mass M is constant, the mass M increases in proportion to the cube of the mass radius R_M, and the Schwarzschild radius R_S, which increases in proportion to the mass M, is immediately larger than the mass radius R_M. Become. Therefore, there can be no supermassive black hole that is expected to exist in each galaxy.

Assuming that the radial density of the mass M is ρ/r, the relationship between the mass radius R_M and the Schwarzschild radius R_S is as follows.

ρ : Radial density coefficient of mass M

ρ/r : Density in the radial direction of mass M

R_M : Mass radius

R_S : Schwarzschild radius

r : Distance from the center of mass M

M_S : Mass when the mass radius R_M and the Schwarzschild radius R_S are equal

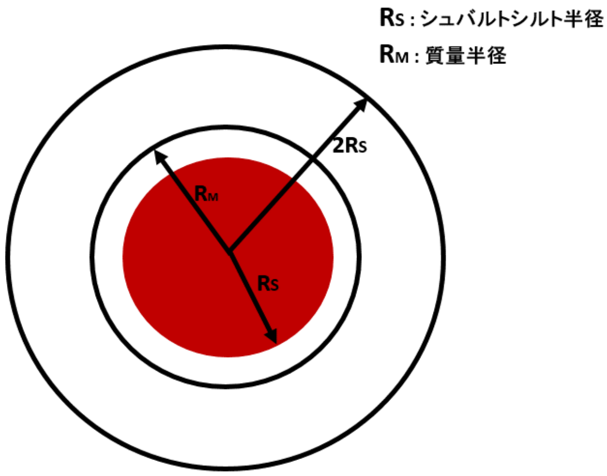
$M = \int_0^{R_M} 4\pi r^2 (\rho/r)\, dr = 2\pi\rho R_M^2 \qquad R_M = \sqrt{M/2\pi\rho}$

$R_S = 2GM/C^2 = 4\pi G\rho R_M^2/C^2$

$R_S = R_M \qquad R_S = C^2/4\pi G\rho \qquad M_S = C^4/8\pi G^2\rho$

As ρ decreases, R_S and M_S increase in inverse proportion to it.

Decrease in ρ allows for the existence of supermassive Black Holes BH_M


RS : シュバルトシルト半径
RM : 質量半径
2RS
RM
RS