Elliptic orbit and Perihelion shift

Hidetomo Tohmori 10/10/2016

A planet moves vertically to cross the distance $\gamma'R$ from the gravitational center of Sun with the constant tangent speed V.

Afterwards, the planet reaches to the distance R' from the gravitational center in the angle θ around Sun. At that time, the advancing direction of the planet is rotating in the angle ω.

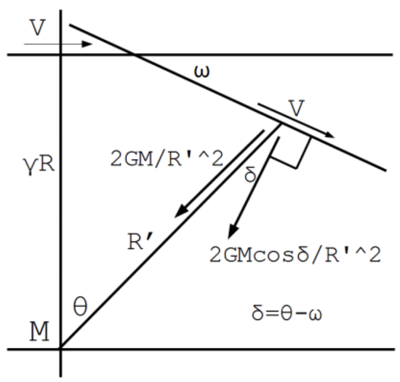

Fig. 1

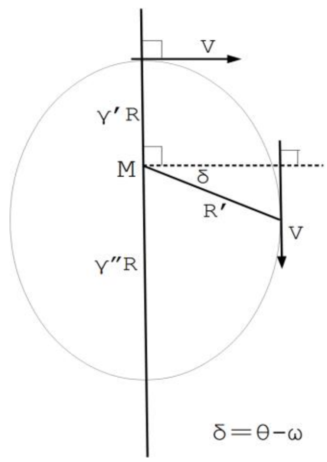

Fig. 2

$\gamma' = 1 - e \quad 2GM/V^2R = 1 \quad U^2 = 2GM/R'$ e: eccentricity

When $\gamma' < 1$, the following expressions hold.

$$V(d\omega/dt) = -(dU^2/dR)\cos\delta = V(d\omega/d\theta)(d\theta/dt)$$

$$d\theta/dt = V\cos\delta/R'$$

$$d\omega/dt = (d\omega/d\theta)(V^2\cos\delta/R') = 2GM\cos\delta/R'^2$$

$$R'\cos\delta = \gamma' R$$

$$d\omega/d\theta = 2GM/V^2R' = \cos\delta/\gamma'$$

$$\delta = \theta - \omega$$

$$d\delta/d\theta = 1 - d\omega/d\theta = (\gamma' - \cos\delta)/\gamma'$$

$$\gamma' = \cos\delta_{\gamma'} \quad d\delta/d\theta = 0 \quad d\omega/d\theta = \cos\delta_{\gamma'}/\gamma' = 1 \quad R' = \gamma'R/\cos\delta_{\gamma} = R$$

When $\gamma" > 1$, the following expressions hold.

$d\theta/dt = V\cos\delta/R'$

$d\omega/dt = (d\omega/d\theta)(V^2\cos\delta/R') = 2GM\cos\delta/R'^2$

$R' = \gamma"R\cos\delta$

$d\omega/d\theta = 2GM/V^2R' = 1/\gamma"\cos\delta$

$\delta = \theta - \omega$

$d\delta/d\theta = (\gamma"\cos\delta - 1)/\gamma"R\cos\delta$

$\gamma"\cos\delta_{\gamma"} = 1 \quad d\delta/d\theta = 0 \quad d\omega/d\theta = 1/\gamma"\cos\delta = 1 \quad R' = R\gamma"\cos\delta_{\gamma"} = R$

$\delta_{\gamma"} = \delta_{\gamma'} \quad \gamma'\gamma" = 1$

$$\omega_{\gamma'} = \int_{-\pi/2-\delta_{\gamma'}}^{\pi/2+\delta_{\gamma'}} d\omega = \int_{-\pi/2-\delta_{\gamma'}}^{\pi/2+\delta_{\gamma'}} (\cos\delta/\gamma')\, d\theta = \int_{-\pi/2-\delta_{\gamma'}}^{\pi/2+\delta_{\gamma'}} ((1-\delta^2)/\gamma')\, d\theta$$

$\cos\delta \cong 1 - \delta^2 \quad \cos\delta_{\gamma'} = \gamma' = 1 - e \quad e$: eccentricity

When $0 < e \ll 1$ i.e. $e^2 \cong 0$, following approximations are effective.

$$\omega_{\gamma'} \cong \int_{-\frac{\pi}{2}-\delta_{\gamma'}}^{\frac{\pi}{2}+\delta_{\gamma'}} (1-\delta^2)\, d\theta (1+e)$$

$$\omega_{\gamma"} \cong \int_{-\frac{\pi}{2}+\delta_{\gamma'}}^{\frac{\pi}{2}-\delta_{\gamma'}} (1-\delta^2)^{-1}\, d\theta (1-e)$$

$(1-e)^{-1} \cong 1 + e \quad (1+e)^{-1} \cong 1 - e \quad (1-\delta^2)^{-1} \cong 1 + \delta^2$

Average of $(1 \pm \delta^2) \cong (2 \pm \gamma'^2)/2 \cong 1 \pm e/2$

$\cos\delta_{\gamma'} \cong 1 - {\delta_{\gamma'}}^2 = 1 - e \;, \quad {\delta_{\gamma'}}^2 \cong e \;, \quad {\delta_{\gamma'}}^4 \cong 0$

$\omega_{\gamma'} \cong (\pi + 2\delta_{\gamma'})(1 - e/2)(1 + e) \cong (\pi + 2\delta_{\gamma'})(1 + e/2)$

$\omega_{\gamma"} \cong (\pi - 2\delta_{\gamma'})(1 + e/2)(1 - e) \cong (\pi - 2\delta_{\gamma'})(1 - e/2)$

$\omega_{\gamma'} \cong \pi(1 + e/2) + 2\delta_{\gamma'}(1 + e/2)$

$\omega_{\gamma"} \cong \pi(1 - e/2) - 2\delta_{\gamma'}(1 - e/2)$

$\omega_{\gamma'} + \omega_{\gamma"} \cong 2\pi + 2\delta_{\gamma'}e$

When the planet moves to draw an elliptic orbit in gravitational fields, the perihelion shift is generated without fail.

	observed value		calculated value
	(second/year)		
Venus	2.04	<	6.087
Earth	11.45	<	14.751
Mars	16.28	<	103.681
Jupiter	6.55		6.132
Saturn	19.50	>	2.899
Uranus	3.34	>	0.835
Neptune	0.36	>	0.0329

Calculated value of planets is within eleven times of observed value. Calculated value of Jupiter is almost corresponding to observed value. It is thought that gravitational field of the other planets doesn't influence Jupiter and oppositely gravitational field of Jupiter influences the other planets. Calculation value is larger than observed value when the orbit of the planet is inside of Jupiter. Oppositely, calculation value is smaller than observed value when the orbit of the planet is outside of Jupiter.