GPS time difference

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The squared time ratio $(dt'/dt")^2$ between GPS satellite and the surface of the earth is obtained as the followings.

$$(iCdt)^2 = (dS')^2 = (iCdt')^2 + (U'dt')^2$$

$$U'^2 = 2GM/R'$$

R' : The distance between GPS satellite and the center of the earth

$R"$: The radius of the earth

dt' : The time passage on GPS satellite

$dt"$: The time passage on the surface of the earth

$$(iCdt)^2 = (dS")^2 = (iCdt")^2 + (U"dt")^2$$

$$U"^2 = 2GM/R"$$

$$(dt'/dt")^2 = (-C^2 + U"^2)/(-C^2 + U'^2)$$

$$= (C^2 - U"^2)(C^2 + U'^2)/(C^4 - U'^4)$$

$$U'^4/C^4 \ll 1 \quad \text{and} \quad U"^4/C^4 \ll 1$$

$$(dt'/dt")^2 = (1 - (1/R" - 1/R')(2GM/C^2))$$

$$= (1 - (1/R" - 1/R')Rs)$$

$$Rs = 2GM/C^2 : \text{Schwarzschild radius}$$

$$R' = \gamma' Rs \ , \quad R" = \gamma" Rs$$

$$(dt'/dt")^2 = (1 - (1/\gamma" - 1/\gamma'))$$

$$(1/\gamma" - 1/\gamma') \ll 1$$

$$dt'/dt" = 1 - (1/2)(1/\gamma" - 1/\gamma')$$

$$dt" - dt' = (1/2)(1/\gamma" - 1/\gamma')dt"$$

$$dt" - dt' = (1/2)(1/R" - 1/R')Rsdt"$$

The time difference $(dt" - dt')$ between the GPS satellite and the surface of the earth occurs during the time $dt"$ on the surface.