Goldbach's conjecture

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1. Generation of two sequences containing every prime number

The formula for the sum of squares of natural numbers is as follows.

$\sum_{i=1}^{k} i^2 = 1 + 2^2 + 3^2 + \cdots + (k-2)^2 + (k-1)^2 + k^2 = k(K+1)(2k+1)/6$

Since the sum of squares of natural numbers is an integer, $k(K+1)(2k+1)$ is divisible by the number 6.

When $p_k = 2k+1$ is a prime number, $(2k+1)$ is indivisible by number 6. At that time, k or $k+1$ is a multiple of the number 3 because either is an even number. m is a natural number.

$$k = 3m \text{ or } k+1 = 3m \qquad p_k = 2k+1 = 6m \pm 1$$

2. Generation of any even number

The following p_m and p_n are composite numbers or prime numbers. n is a natural number.

$$p_m = 6m \pm 1 \qquad p_n = 6n \pm 1 \qquad m + n = l$$

$$\begin{aligned} p_m + p_n = 6m \pm 1 + 6n \pm 1 &= 6(m+n) - 2 = 6l - 2 \\ &= 6(m+n) \qquad = 6l \\ &= 6(m+n) + 2 = 6l + 2 \end{aligned}$$

As mentioned above, any even number greater than or equal to 10 ($l \geq 2$) can be generated by the sum of p_m and p_n.

Using prime number theorem "probability that a natural number x is a prime number $1/logx$", the probability q that there is a combination in which both p_m and p_n are prime numbers is as follows.

$$q = \sum_{m=1}^{l/2} \left(\frac{3}{\log(6m)}\right)\left(\frac{3}{\log(6(l-m))}\right) \cong \frac{9l}{2(logl)^2}$$

This probability q diverges infinitely as l increases.

Goldbach conjecture seems to hold, of course.

Prime number theorem is proved based on Riemann hypothesis.

However, Riemann hypothesis has not yet been proved.

Then, when Riemann hypothesis does not hold, prime number theorem does not hold, and Goldbach's conjecture may not hold.