Infinite existence of Sophie Germain primes

Hidetomo Tohmori 2022/11/15

1. At the beginning

Sophie Germain prime p is also prime $2p$+1. The infinite existence of such primes is proved by the prime number theorem. And the prime number theorem is easily proved.

2. Generation of two sequences containing every prime number

The formula for the sum of squares of natural numbers is as follows.

$$\sum_{i=1}^{k} i^2 = 1 + 2^2 + 3^2 + \cdots + (k-2)^2 + (k-1)^2 + k^2 = k(K+1)(2k+1)/6$$

Since the sum of squares of natural numbers is an integer, $k(K+1)(2k+1)$ is divisible by the number 6.

When $p_k = 2k+1$ is a prime number, $(2k+1)$ is indivisible by number 6. At that time, k or $k+1$ is a multiple of the number 3 because either is an even number. m is a natural number.

$k = 3m$ or $k+1 = 3m$ $\quad$ $p_m = 2k+1 = 6m \pm 1$

3. Infinite existence of Sophie Germain primes

Sophie Germain prime p_m is also prime $p_n = 2p_m + 1$

$p_m = 6m - 1 \quad p_n = 12m - 1$

Using the prime number theorem "The probability that a natural number x is prime is $\frac{1}{\ln x}$", the probability q that both p_m and p_n are prime is as follows.

$$q = \sum_{m=1}^{l} \left(\frac{1}{\ln(6m-1)}\right)\left(\frac{1}{\ln(12m-1)}\right) \geq \frac{l}{(\ln l)^2}$$

It is known that $\frac{l}{(\ln l)^2}$ on the right side diverges to infinity as l increases.

Then, this probability q diverges infinitely as l increases.

Therefore, there are infinitely many Sophie-Germain primes p_m.

4 A proof of the prime number theorem

At the beginning

When a natural number p is a prime number p, the prime number p exists infinitely in all natural numbers in the form of power p^n.

However, the existence probability $\frac{1}{\ln p}$ of power p^n in all natural numbers can be obtained from the existence probability of power p^n in natural number e^m.

And the existence probability $\frac{1}{\ln p}$ of the power p^n is the existence probability $\frac{1}{\ln p}$ of the prime number p.

Then every natural number is prime p with existence probability $\frac{1}{\ln p}$.

That is, natural number p is prime number p with the existence probability $\frac{1}{\ln p}$.

That is, the existence probability $\frac{1}{\ln p}$ is the probability that natural number p is prime.

The prime counting function $\pi(x)$ is obtained by integration with probability $\frac{1}{\ln p}$.

Derivation of $\frac{1}{\ln p}$

There are always natural numbers n and m that hold the following inequality for every prime number p.

e is Napier's constant

$$e^m < p^n < e^{m+1}$$

Converting to logarithm, the following inequality holds.

$$m < n \ln p < (m+1) \qquad \ln e = 1$$

$$1 < \frac{n \ln p}{m} < (1 + \frac{1}{m})$$

When the natural number m goes to infinity, the following equality holds.

$$\lim_{m\to\infty}\frac{n\ln p}{m}=1$$

$$\lim_{m\to\infty}\frac{n}{m}=\frac{1}{\ln p}$$

Power p^n and power e^m exist in natural numbers e^m as follows.

$$e^m < p^n < e^{m+1}$$

$$\begin{array}{r}
{p_l}^1 \\
{p_{l-1}}^1\ {p_{l-1}}^2 \\
{p_{l-2}}^1\ {p_{l-2}}^2\ {p_{l-2}}^3 \\
\dots\ \dots\ \dots\ \dots\ \dots \\
p^1\ p^2\ p^3 \dots\ \dots\ \dots\ \dots\ \dots\ p^n\ p^{n-1}\ p^n \\
\dots\ \dots\ \dots\ \dots\ \dots \\
{p_3}^1\ {p_3}^2\ {p_3}^3\ \dots\ \dots\ \dots\ \dots\ \dots\ \dots\ \dots\ {p_3}^{j-2}\ {p_3}^{j-1}\ {p_3}^j \\
{p_2}^1\ {p_2}^2\ {p_2}^3\ {p_2}^4 \dots\ \dots\ \dots\ \dots\ \dots\ \dots\ \dots\ \dots\ {p_2}^{k-3}\ {p_2}^{k-2}\ {p_2}^{k-1}\ {p_2}^k \\
{p_1}^1\ {p_1}^2\ {p_1}^3\ {p_1}^4\ {p_1}^5\ \dots\ \dots\ \dots\ \dots\ \dots\ \dots\ \dots\ {p_1}^{i-4}\ {p_1}^{i-3}\ {p_1}^{i-2}\ {p_1}^{i-1}\ {p_1}^i \\
e^1\ e^2\ e^3\ e^4\ e^5\ e^6\ \dots\dots\ \dots\dots\ \dots\dots\ \dots\dots\ e^{m-5}\ e^{m-4}\ e^{m-3}\ e^{m-2}\ e^{m-1}\ e^m
\end{array}$$

Therefore, the exponential ratio $\frac{n}{m}$ is the existence probability of prime p among the natural numbers e^m.

Then $\lim_{m\to\infty}\frac{n}{m}$ is the existence probability $\frac{1}{\ln p}$ of prime p among all natural numbers.

And since the prime p exists among all natural numbers with the existence probability $\frac{1}{\ln p}$, every natural number is prime p with the existence probability $\frac{1}{\ln p}$.

That is, the existence probability $\frac{1}{\ln p}$ is the probability that natural number p is prime.

Derivation of the prime counting function $\pi(x)$

The prime counting function $\pi(x)$ is obtained by integration with the probability $\frac{1}{\ln p}$ as follows.

$$\pi(x) = \int_2^x \left(\frac{1}{\ln p}\right) dp \cong x/\ln x$$

Thus, the prime number theorem has been proved.

References

1. Erdős, Paul (1949-07-01), "On a new method in elementary number theory which leads to an elementary proof of the prime number theorem,", Proceedings of the National Academy of Sciences (U.S.A.: National Academy of Sciences) 35 (7): 374-384, doi:10.1073/pnas.35.7.374