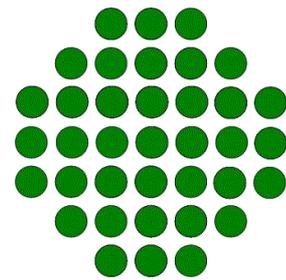
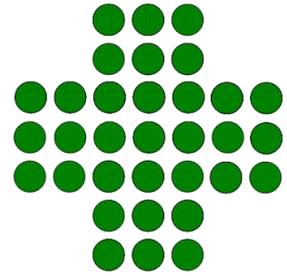


# Peg Solitaire British

**Hana:** Peg Solitaire Diamond (Forty-One) is one of Peg Solitaires.

**Taro:** It is so. However, almost people think that only Peg Solitaire British shown in right figure is Peg Solitaire, because it is the most popular.



**Taro:** Peg Solitaire French shown in left figure is known well too.

**Hana:** How is Peg Solitaire British played ?

**Taro:** The way to advance the game is quite the same also in any.

Solitaire Diamond shown in a right figure.

**Taro:** Peg Solitaire British has not eight White stones and holes of Peg

**Hana:** It seems to be easy to end the game.

**Taro:** It is so.

**Hana:** Do you know the reason?

**Taro:** Let's examine the reason here.

**Taro:** Either of First stone and it's equivalent stones becomes Last stone

in Peg Solitaire British.

**Hana:** Please teach it to me neatly.

**Taro:** I have explained the reason at Peg Solitaire Diamond.

**Hana:** However, I don't remember at all.

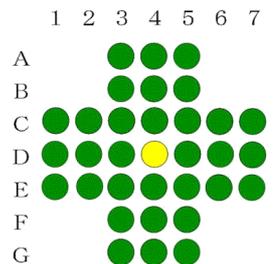
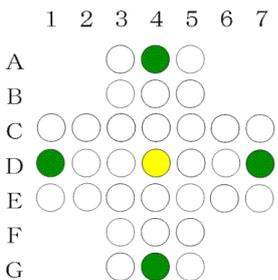
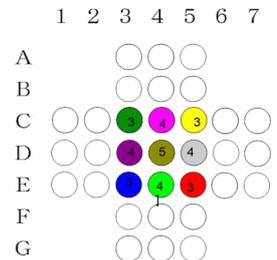
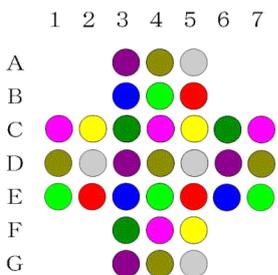
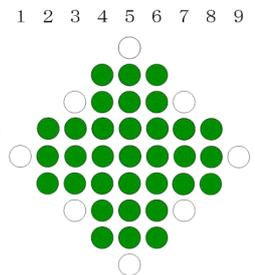
**Taro:** The stone doesn't remain anywhere as shown in left figure when the game is played without removing First stone by Virtual rule in Peg Solitaire British.

**Hana:** What does it mean?

**Taro:** It means that First stone or it's equivalent stone becomes Last stone when the game is played by removing First stone.

**Taro:** For instance, when Yellow stone D4 is First stone as shown in a right figure, either of stone D4, A4, D1, D7 or G4 only remains by Virtual rule.

**Hana:** By the way, what is Virtual rule?



Taro: Have you already forgotten? Then, let's review Virtual rule.

Taro: According to Virtual rule, two or more stones may enter into one hole.

Taro: However, if the number of stones in a hole is even, the hole is the same as "no stone", and if the number is odd, the hole is the same as "one stone".

Taro: Therefore, for example, when one stone is moved from Hole A to Hole B stepping over Hole C, Hole A with "one stone" becomes "no stone", and Hole A with "no stone" becomes "one stone". And also, Holes B and C are the same as Hole A.

Taro: In Holes A,B and C, even number changes to odd number and odd number changes to even number.

Hana: It looks like EXOR (Exclusive OR) logic operation.

$$1+1=1 \quad 1+0=1 \quad 0+1=1 \quad 0+0=0$$

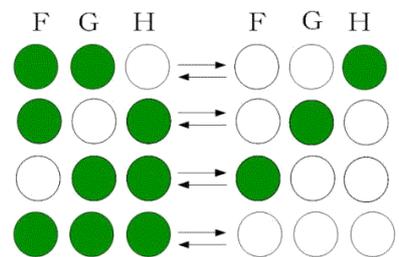
Taro: Exactly, it is so.

Taro: This operation is called "mode 2 surplus operation". Therefore, when the number to be divided by two is even, it changes to 0, and when the number is odd, it changes to 1.

Taro: It is possible to play the game without removing First stone by Virtual rule though it is impossible by Usual rule.

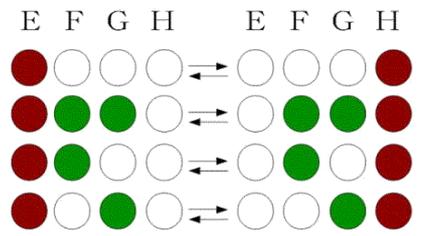
Hana: It was so. I have gradually recalled it.

Taro: When the game is done by Virtual rule, the conversion of the array of stones and the transfer of stone are possible as shown in a right figure.



Hana: In Peg Solitaire British, the game is rather difficult because there is no limitation different from Peg Solitaire Diamond 41.

Taro: That exactly. It is easy to play the game in Peg Solitaire Diamond and Solitaire French compared with Peg Solitaire British because the movements of stones are limited.



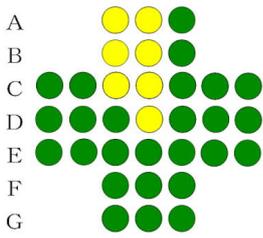
Taro: Then, let's examine how to play Peg Solitaire British.

Hana: Indeed, I wanted to hear it in reality.

Taro: In Peg Solitaire British too, Last stone is predictable from First stone by Virtual rule.

Hana: I think that it is easy to play Peg Solitaire British if Last

stone is predictable from First stone.



**Taro:** Considering the symmetry of Peg Solitaire British, First stone is either of seven stones A3, A4, B3, B4, C3, C4, and D4 shown in left figure.

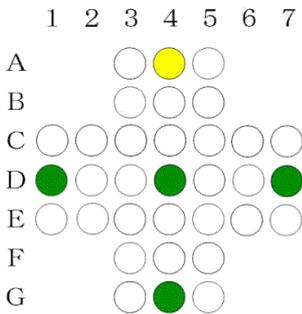
**Hana:** Because other stones are symmetrical with either of the above mentioned seven Yellow stones, we need not to examine the other stones.

**Taro:** Further more, since stone A3 is equivalent to stone D6, stone D6 is symmetrical with stone B4, stone B4 is equivalent to stone E4, and stone E4 is symmetrical with stone C4, we need to examine combination patterns of stones A3, B4 and C4.

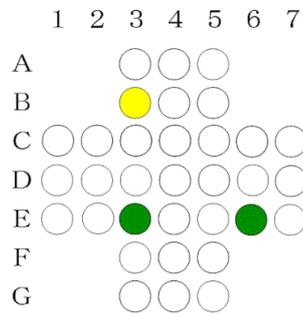
**Hana:** If so, since stone D4 is equivalent to stone A4, we need to examine combination pattern of stones D4 and A4. And, since stone B3 is equivalent to stone E3, and stone E3 is symmetrical with stone C3, we need to examine combination pattern of stones B3 and C3.

**Taro:** Therefore, we need to verify following three combination Patterns (1), (2) and (3).

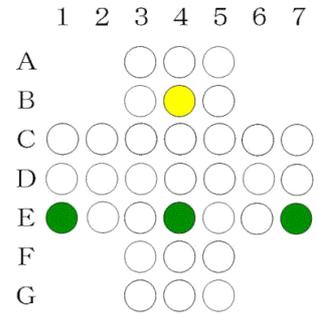
(1)



(2)



(3)



**Taro:** First of all, let us verify Pattern (1).

**Hana:** Then, may I question on Pattern (1).

**Taro:** What is it?

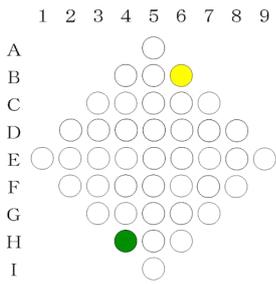
**Hana:** For instance, when stone A4 is First stone, does stone A4 really become Last stone without fail?

**Hana:** Because Usual rule is different from Virtual rule.

**Taro:** Certainly, it is so.

**Taro:** However, I think that it is very difficult to prove it to be always possible.

**Hana:** In Peg Solitaire Diamond 41, even if the game is able to be ended



by Virtual rule, the game might not necessarily be ended by Usual rule.

**Taro:** When we proved that the game of First stone B6 and Last stone H4 in Peg Solitaire Diamond is impossible, we examined all mobility patterns of all stones.

**Taro:** In Peg Solitaire Diamond, because the number of mobility patterns was little, it did not take time too much to examine all the mobility patterns.

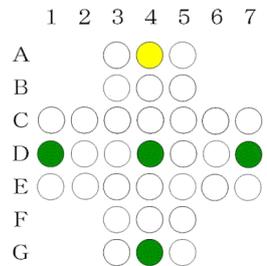
**Taro:** However, because there is no limitation relating to the movement of stones in Peg Solitaire British, the number of the mobility patterns of stones is extremely large.

**Hana:** Even if it is so, do not you know any methods?

**Taro:** Anyway, let's examine whether an answer actually exists about each of combinations of First stone and Last stone.

**Taro:** First of all, let's think about the combination of Pattern 1 in which stone A4 is First and Last stone.

**Hana:** Why is such a thing done? Has the answer to stone A4 already gone out?



**Taro:** Certainly, we have already understood that stone D4 becomes First and Last stone.

**Taro:** However, we have not understood yet that stone A4 becomes First and Last stone.

**Taro:** stone A4 might not be able to become First and Last stone.

**Hana:** Certainly, that is so.

**Taro:** Let's examine whether stones A4, D1, D7 and F4 become Last stone, when stone A4 is First stone by Usual rule.

**Hana:** Can you prove them?

**Taro:** It is thought without fail that the proof can be substituted by showing one actually possible example even if the proof is impossible.

**Hana:** Using the computer, we can judge whether the combination is possible or impossible.

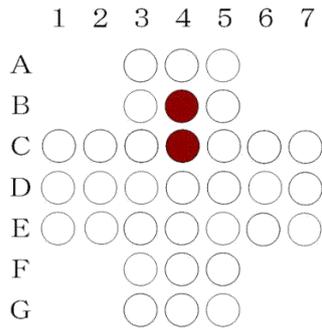
**Taro:** However, the computer answer nothing about why it is possible or impossible.

**Hana:** Let us too examine the combination apart from why possible.

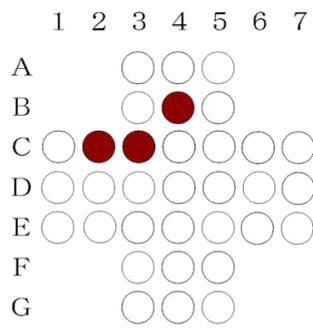
**Taro:** Do not you want to understand why it to be possible? Even if it is a God, you cannot be saved.

**Taro:** Apart from it, when stone A4 is Last stone, the array of last two stones, last three stones, and last four Red stones are shown in following figures.

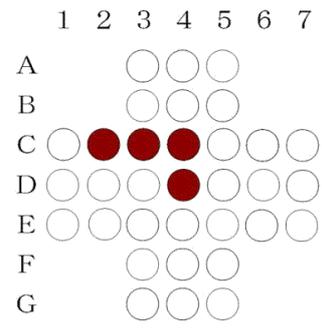
Last two Red stones



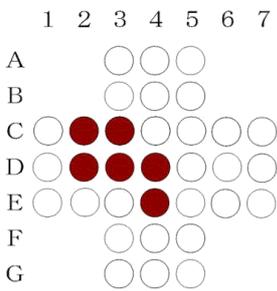
Last three Red stones



Last four Red stones



Last six Red stones



Taro: In addition, we think about last six Red stones as shown in a left figure.

Hana: I think that there are many cases besides the case of "last six Red stones" as shown in the left figure .

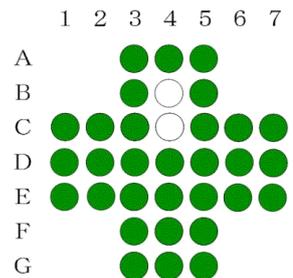
Taro: Anyway, let us examine only the case as shown in the left figure.

Hana: How forcible you are!

Taro: Because we cannot examine every case.

Hana: You should take up the case only to examine it carefully.

Taro: Necessarily, it is not so. I selected the case because it seemed to be possible.



Hana: So, I cannot help you.

Taro: Then, If Green stones in right figure are replaced with the last six Red stones, it becomes as shown in left figure.

Hana: Why do you replace Green stones in a right figure with the last six Red stones?

Taro: It is because the Green stones that should be removed are made comprehensible.

Hana: Why is the left figure comprehensible?

Taro: If you carefully observe the left figure, you can understand it naturally.

Hana: Indeed, all Green stones of the left figure only has to be removed.

Taro: Though it is good in it.

Hana: You are no vigour.

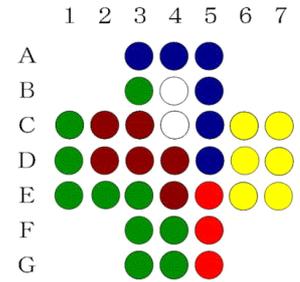
Taro: I can not see ahead.

Hana: Are you worried about that you do not know how to divide all the Green stones by every three stones arrayed vertically or horizontally.

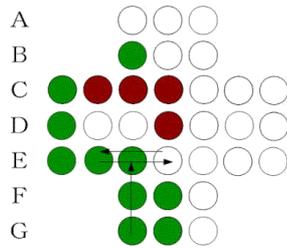
Taro: Anyway, let's try.

Hana: Where do you start?

Taro: the Blue stones, the Yellow stones, and the Bright Red stones in right figure are easily removed.



1 2 3 4 5 6 7 Then, the number of Red stone's is changed from six to four as shown in left figure.

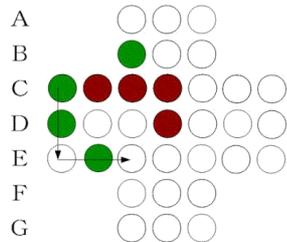


Hana: How is Green stone B3 removed?

Taro: Anyway, it becomes right figure when executing Green stone  $E2 \rightarrow E4$ , Green stone  $G3 \rightarrow E3$  and Green stone  $E4 \rightarrow E2$ .

Hana: It only puts the problem off.

1 2 3 4 5 6 7 Taro: Oh dear, please look.

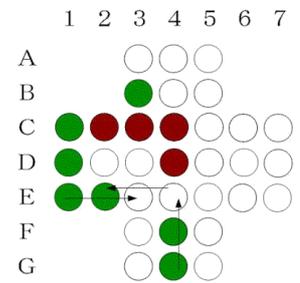


Hana: How do you do?

Taro: Green stone  $E1 \rightarrow E3$  and Green stone  $G4 \rightarrow E4 \rightarrow E2$  are executed as shown in right figure.

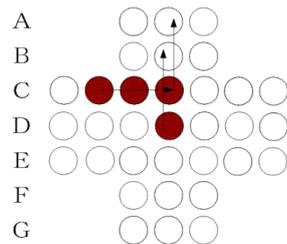
Hana: And, do?

Taro: Green stone  $C1 \rightarrow E1 \rightarrow E3$  is executed as shown in left figure.



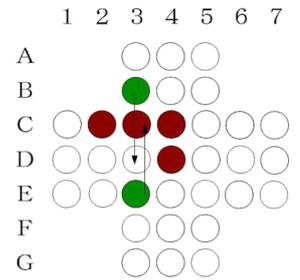
Hana: This is helpless.

1 2 3 4 5 6 7 Taro: However, two Green stones B3 and E3 are removed if Green stone  $B3 \rightarrow D3$  and Green stone  $E3 \rightarrow C3$  are executed as shown in right figure.



Hana: I seem to be cheated.

Taro: By this example, it has been proven that stone A4 become First and Last stone.



Last stone.

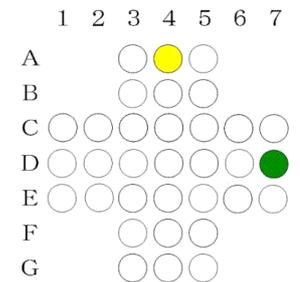
Hana: Then, when stone A4 is First stone, can either of stones D1 and D7 become Last stone ?

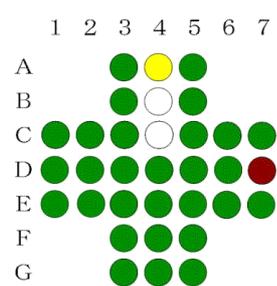
Taro: It can not be proven.

Taro: However, it is well known that it is actually possible by Usual rule.

Taro: Because it is easy, let's actually do and show it.

Hana: It is good.



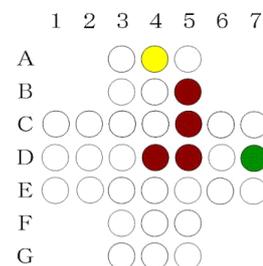


Taro: First of all, let's try the case where the stone D7 becomes Last stone.

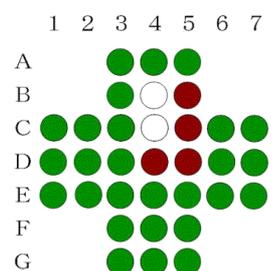
: When stone C4 is moved to A4, it becomes left figure.

Hana: The First stone movement has only it.

Taro: If Last stone is Green stone D7, four Red stones as shown in a right figure are sure to remain.



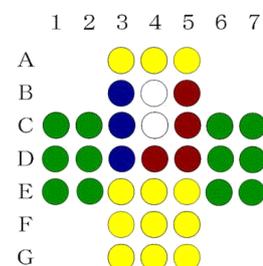
Hana: Why do you think about such a thing?



Taro: If Green stones in left figure are replaced with four Red stones as shown in the right figure, it becomes left under figure.

Hana: And then, what will you do from it?

Taro: If all the Green stones can be removed in left figure, last four Red stones in right figure only remain, and then, Last stone will surely remain in hole D7.

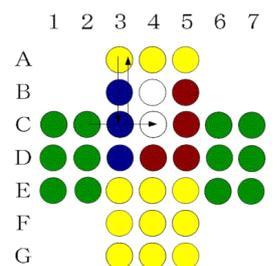


Hana: Though it is so.

Taro: Then, all the Green stones are divided by every three stones arrayed vertically or horizontally as shown in right figure.

Hana: Why do you divide by every three stones?

Taro: The reason is that the arrayed every three stones are easily removed.



Hana: Isn't Usual rule so though it might be so in Virtual rule?

Taro: Even Usual rule is so.

Hana: Please teach why Usual rule is so plainly.

Taro: All the Green stones have been divided by every three stones.

Hana: After that, how do you do?

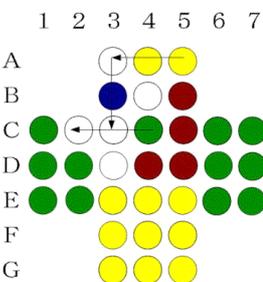
Taro: First of all, horizontal Yellow stones A3, A4 and A5 and vertical Blue stones B3, C3 and D3 are removed together.

Hana: Please teach me it plainly.

Taro: The technique is necessary.

Taro: First of all, Green stone C2 → C4, Yellow stone A3 → C3 and Blue stone D3 → B3 are executed as shown in left figure.

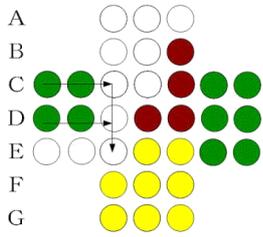
Taro: In addition, Yellow stone A5 → A3, A3 → C3 and C4 → C2 are



executed as shown in left figure.

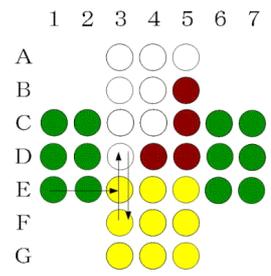
**Hana:** All of **Yellow** stones A3 , A4, A5 and **Blue** stones B3, C3, D3 have disappeared.

**Taro:** Next, six **Green** stones C1, C2, D1, D2, E1 and E2 are removed.



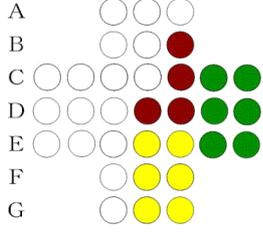
**Taro:** First of all, **Yellow** stone F3 → D3, **Green** stone E1 → E3 and **Yellow** stone D3 → F3 are executed as shown in right figure.

**Taro:** In addition, **Green** stone C1 → C3, **Green** stone D1 → D3 and **Green** stone C3 → E3 are executed as shown in left figure.



**Hana:** Mysterious! Only six **Green** stones have disappeared.

**Taro:** Next, nine **Yellow** stones from E3 to G5 are removed.



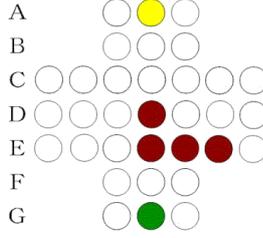
**Taro:**First of all, **Yellow** stone E4 → E2, **Yellow** stone G3 → E3, and **Yellow** stone E2 → E4 are executed.

**Hana:** Six remaining **Yellow** stones can be removed by repeating this, too.

**Taro:** It is so.

**Taro:** Six **Green** stones from C6 to E7 can be removed by the technique similar to six **Green** stones ahead.

**Hana:** Only last four **Red** stones remain.



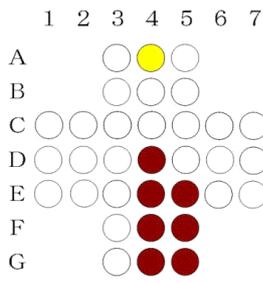
**Taro:** As a result, it has been actually shown that stones D1 or D7 can become Last stone when stone A4 is First stone.

**Hana:** The case of First stone A4 and Last stone G4 is still not examined.

**Taro:** Let's try to examine it as follows.

**Hana:** At first, how do you do?

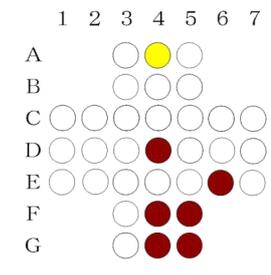
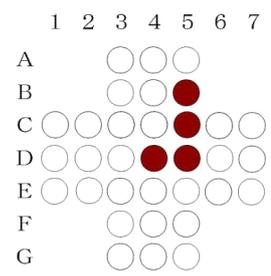
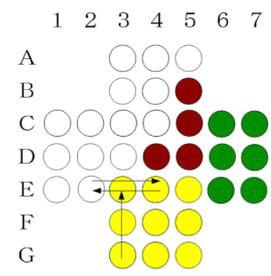
**Taro:** To divide all the **Green** stones by every three stones arrayed vertically or



horizontally, the last four **Red** stones are preprocessed as follows.

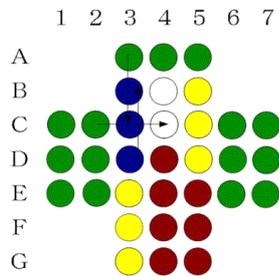
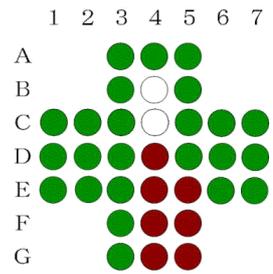
**Taro:** Up to now, the last four **Red** stones have been used as shown in upper left figure.

**Hana:** It is so.



Taro: At this time, last seven Red stones as shown in left picture are used.

Hana: It is necessary to remove all Green stones in right picture such that the last seven Red stones only remain.



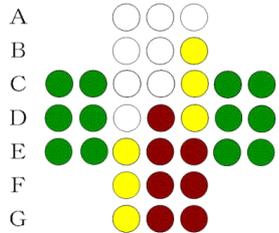
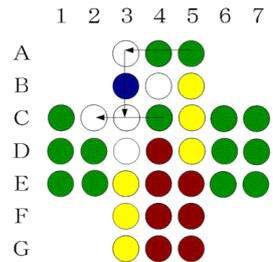
Taro: It is so.

Taro: All Green stones are classified into three colors by every three stones as shown in left figure.

Hana: Then, how do you do?

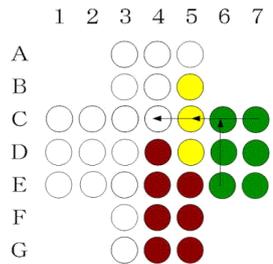
Taro: It is not difficult so much.

Taro: First of all, Green stone C2 → C4, Green stone A3 → C3, and Blue stone D3 → B3 are executed as shown in left figure.

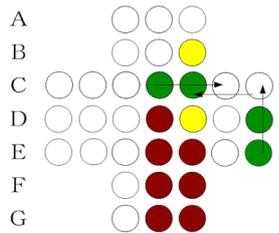


Taro: In addition, Green stone A5 → A3, Green stone A3 → C3, and Green stone C4 → C2 are executed as shown in right figure.

Hana: Green stones A3, A4, A5 arranged horizontally and Blue stones B3, C3, D3 arranged vertically were removed all together.



Taro: Six Green stones C2 to E2 and three Yellow stones E3 to G3 are removed by the similar technique used before.

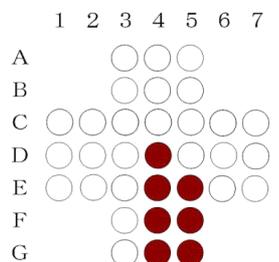
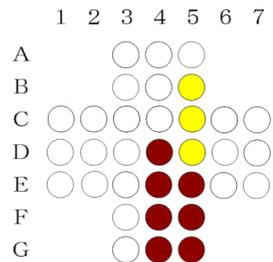


Hana: How are three Yellow stones B5 to D5 removed?

Taro: The technique is necessary

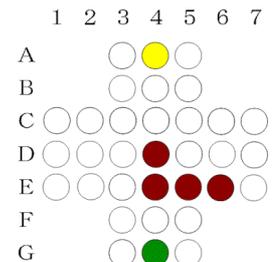
Hana: How do you do?

Taro: First of all, Green stone C6 → C4, Green stone E6 → C6, and Green stone C7 → C5 are executed.



Taro: In addition, Green stone C4 → C6, Green stone E7 → C7, and Green stone C7 → C5 are executed. Then, six Green stones C6 to E7 are removed as shown in right picture.

Hana: Green stone E7 is changed into Yellow stone C5.



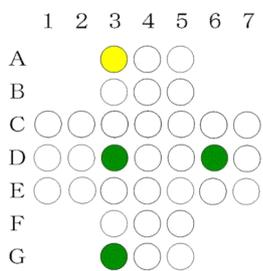
Taro: Three Yellow stones D5 to B5 in any way has not been changed.

Taro: Then the three Yellow stones B5, C5 and D5 are

easily removed.

Hana: Indeed, Last stone certainly remains in hole G4.

Taro: This is an example that Green stone G4 become Last stone when Yellow stone A4 is First stone.



Hana: At last, the combination Pattern (1) ended.

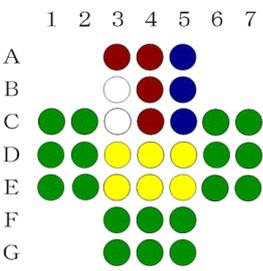
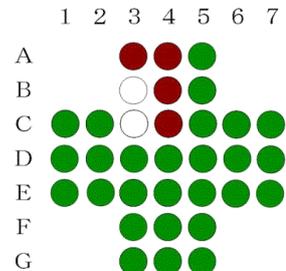
Taro: In any order, let's examine the combination Pattern (3).

Hana: Can Yellow stone A3 become First and Last stone?

Taro: Let's examine it.

Hana: At this time too, last four Red

stones are used to examine the case.



Taro: It is so. The right figure is examined.

Hana: In the right figure, all the Green stones have to be removed.

Taro: It is so.

Hana: To remove all the Green stones, they are classified into colors by every three stones arrayed

vertically or horizontally.

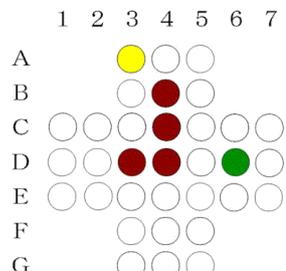
Taro: It is shown in left figure.

Hana: If this is so, even I can remove all the Green stones.

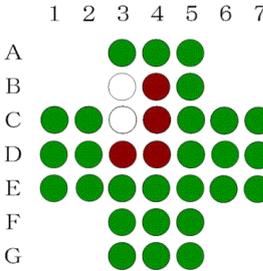
Taro: stone A3 easily becomes First and Last stone.

Hana: Then, when stone A3 is First stone, can stone D6 become Last stone ?

Taro: In the case of First stone A3 and Last stone D6, last four Red stones are arrayed as shown in right figure.



Hana: Also, Green stones are replaced with the last four Red stones as shown in left figure.

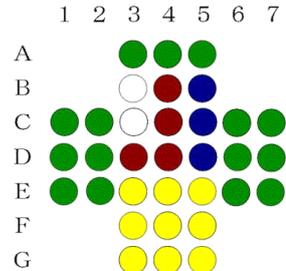


Taro: If all the Green stones in right picture are removed, D6 can become the Last stone.

Hana: Let's classify all Green stones into colors by every three stones vertically or horizontally.

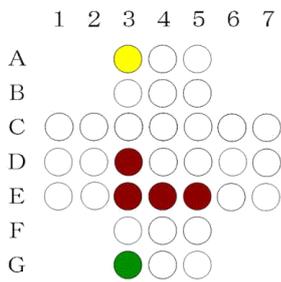
Taro: In this case too, all the Green

stones are easily removable.



Hana: At first, six Green stones on the left side are removed. Next, nine Yellow stones are removed. Then six Green stones on right side are

removed. And at last, three Green stones arrayed horizontally in upper part and three Blue stones arrayed vertically are removed together.



Taro: Did you understand well?

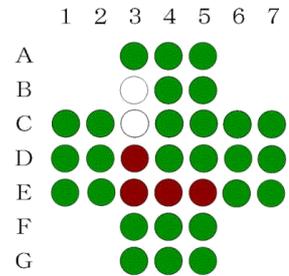
Hana: Last stone may remain in Hole D3 too.

Taro: Because stone D3 is equivalent to stone D6

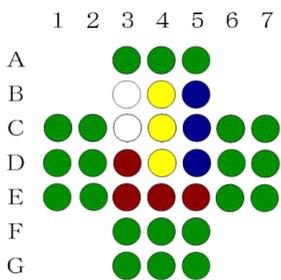
Hana: By the way, can stone G3 become Last stone when stone A3 is First stone?

Taro: If stone G3 is Last stone,

last four Red stones ought to be arrayed as shown in left figure.



Hana: Green stones to be removed are arrayed with the last four Red stones as shown in right figure.



Taro: Therefore, if all the Green stones in right figure can be removed, stone G3 can become Last stone.

Hana: Let us classify all the Green stones into colors by every three stones arrayed vertically or horizontally.

Taro: At first, three Yellow stones are removed.

Next, three Green stones of upper part are removed together with vertical three Blue stones. Then, six Green stones of left side are removed. Further, six Green stones of lower side are removed. And at last, six Green stones of right side are removed.

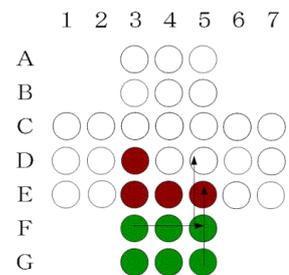
Hana: Please show me how the six Green stones may be removed.

Taro: I have shown you the same thing before.

Hana: I forgot, and show it again, please.

Taro: The technique is necessary.

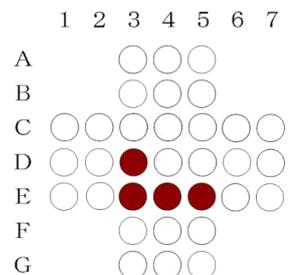
Hana: Please show it early without irritating it.



Taro: At first, Green stone  $F5 \rightarrow D5$ , Green stone  $F3 \rightarrow F5$  and Green stone  $G5 \rightarrow E5$  are executed as shown in right figure.

Hana: After that, how do you do?

Taro: In addition, Green stone  $D5 \rightarrow F5$ , Green stone  $G3 \rightarrow G5$ , and Green stone  $G5 \rightarrow E5$  are executed as shown

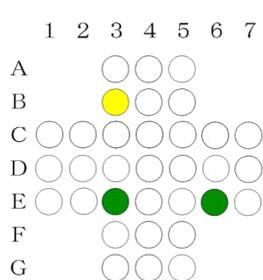


in left figure.

Hana: Indeed.

Taro: Green stone G3 replaces Red stone E5.

Taro: I seem to have forgotten something though I don't know what it



is.  
**Hana:** It is so, we have not examined the pattern of left figure yet.

**Taro:** That is the combination pattern (2).

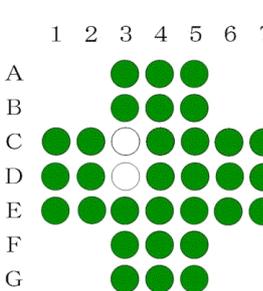
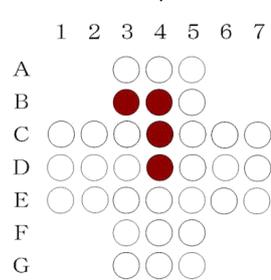
**Hana:** Because stone E6 is symmetrical with B3, it is nesessary to examine stones B3 and E3.

**Taro:** When stone E3 is First stone,

Last stone is either stones E3, B3 and E6.

**Hana:** When stone E3 is First stone, stone E3 may become Last stone.

**Taro:** Therefore, it is necessary to examine whether stone B3 can actually become First and Last stone.



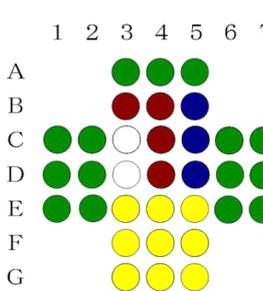
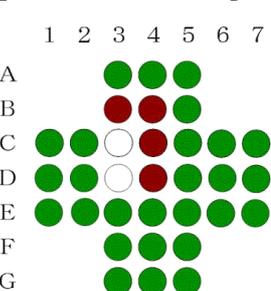
**Hana:** Indeed.

**Taro:** When stone B3 is First and Last stone, last four Red stones remain as shown in right figure.

**Hana:** When left figure is overlapped with right figure, it become lower right figure.

**Taro:** And, all Green stones are classified into colors by every three stones arranged vertically or horizontally as shown in right figure.

**Hana:** According to right figure, it seems easy to remove all the Green stones.

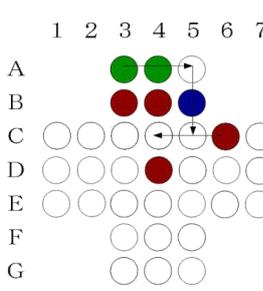


**Taro:** It is just, all the Green stones except the last four Red stones can be easily removed.

**Hana:** Though vertical three Blue stones can be removed, how can horizontal three Green stones of upper part be removed?

**Taro:** I remember the same as this.

**Hana:** I have forgotten.



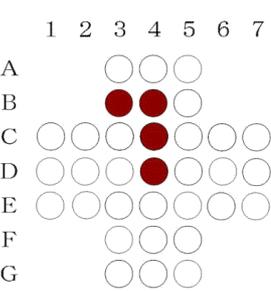
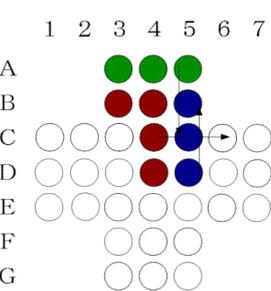
**Taro:** The technique is necessary.

**Hana:** Please show me it again.

**Taro:** The vertical three Blue stones and the horizontal three Green stones of upper part need to be removed together.

**Hana:** How do you actually do?

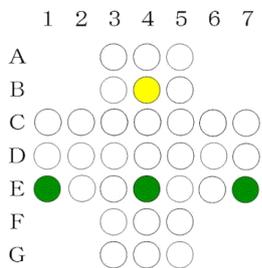
**Taro:** First, Red stone C4 → C6, Green stone A5 → C5,



and Blue stone D5 → B5 are executed.

Hana: Next, how do you do?

Taro: After that, Green stone A3 → A5, Green stone A5 → C5, and Red stone C6 → C4 are executed.



Hana: It is wonderful!

Taro: Thus, by this example, it was shown that Red stone B3 became First and Last stone.

Hana: Then, let us examine combination pattern (3).

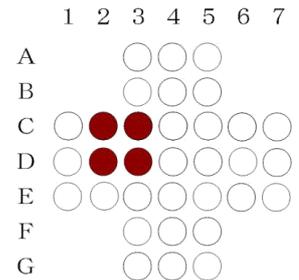
Taro: The combination pattern (3) is shown in left figure.

Hana: It is so. Please show the example that stone B4 is First and Last stone.

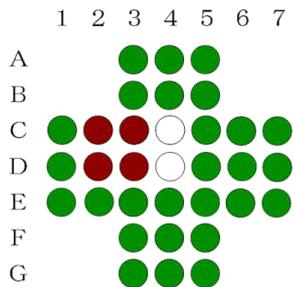
Taro: I think that the case of stone B4 is easier than the case of stone A4.

Hana: Anyway, please show me the example.

Taro: If stone B4 become Last stone, last four Red stones need to remain as shown in right figure.



Hana: Left figure shows the case when Green stones after First movement are replaced with last four Red stones.



Taro: Green stones of left figure are classified into colors by three stones arrayed vertically or horizontally as shown in right figure.

Hana: In this case, it seems easy to leave the last four Red stones.

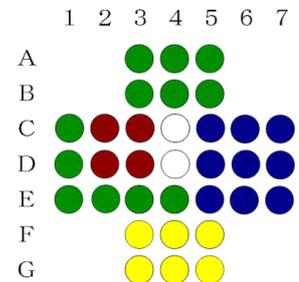
Taro: It is so.

Hana: What have you understood from the above-mentioned examples after all?

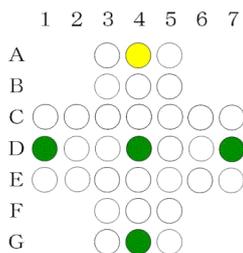
Taro: It can be summarized to the following two points.

Taro: 1. There are three kinds of combination Patterns (1) to (3).

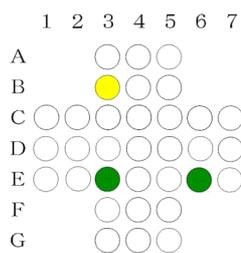
Taro: 2. Any combination Pattern of First stone and Last stone is actually possible.



(1)



(2)



(3)

