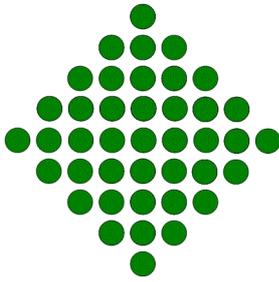
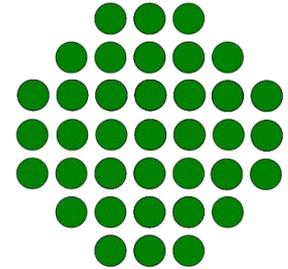


Peg Solitaire French



Hana: Peg Solitaire Diamond shown in left figure is one of Peg Solitaire games.

Taro: It is so. There are Peg Solitaire French shown in right figure and Peg Solitaire British shown in lower right figure, too.



British.

Taro: Only it is seen.

Taro: In the Internet, Most famous Peg Solitaire is British board.

Hana: How is the game advanced in Peg Solitaire French ?

Taro: If how to advance Peg Solitaire Diamond is understood, it doesn't have very.

Hana: Maybe, I also think it is so.

Hana: How is Peg Solitaire French different from Peg Solitaire Diamond ?

Taro: Then, let's try to examine Peg Solitaire French as well as Peg Solitaire British.

Hana: Peg Solitaire French is addition of four stones B2, B6, F2, and F6 to Peg Solitaire British.

Hana: It seems difficult to play the game of Peg Solitaire French.

Taro: It is actually so.

Hana: Do you know the reason?

Taro: I am thinking to examine the reason.

Taro: The number of stones of Peg Solitaire French are four stones more than that of Peg Solitaire British.

Taro: And, Peg Solitaire French has 16 First stones though Peg Solitaire British has 7 First stones.

Hana: How do you understand this?

Taro: You may understand that well if you play Peg Solitaire French by Virtual rule.

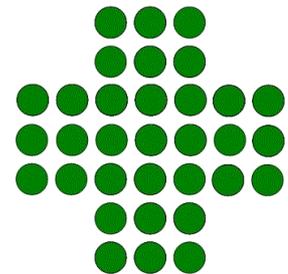
Hana: By the way, what is Virtual rule?

Taro: Then, let's review Virtual rule.

Taro: In Virtual rule, there are two or more stones in a hole.

Taro: However, it is assumed that there is no stone in the hole if the number of stones of the hole is even, and that there is one stone in the hole if the number is odd.

Taro: As well as Usual rule, one stone is moved by stepping over



adjacent hole and then one stone of the adjacent hole is removed.

Taro: However, it is possible to move the stone by stepping over the adjacent hole with no stone, and then the adjacent hole becomes the hole with one stone.

Taro: At that time, the moving destination hole becomes the hole with one stone if it is the hole with no stone, and becomes the hole with no stone if it is the hole with one stone.

Taro: When paraphrasing it, there are even number stones in the hole with no stone and odd number stones in the hole with one stone.

Hana: It looks like XOR (Exclusive OR) somehow.

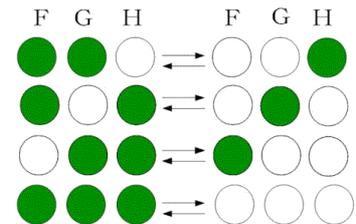
$$1+1=1 \quad 1+0=1 \quad 0+1=1 \quad 0+0=0$$

Taro: Exactly, it is so.

Taro: It is said "mode 2 surplus operation" of two (Only the remainder when dividing by two is assumed to be a result of the operation. The even number is 0, and, therefore, the odd number is 1).

Hana: Where is different from Usual rule as for Virtual rule?

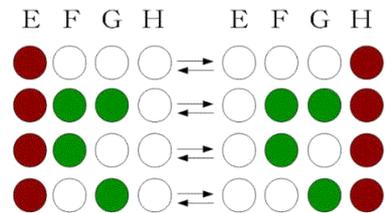
Taro: Conversion as shown in right figure becomes possible in Virtual rule.



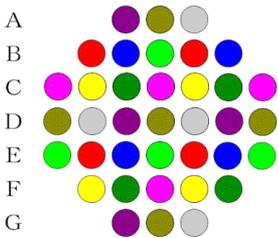
Taro: And, it is possible to play the game without removing First stone by Virtual rule.

Hana: How is the game advanced?

Taro: In the application of the conversion of the right as it is, the game becomes complex.



1 2 3 4 5 6 7

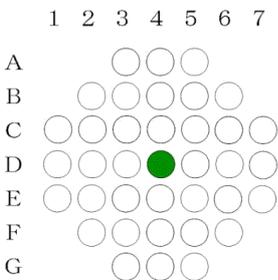


Hana: Then, how should I do?

Taro: 37 stones of Peg Solitaire French are classified into nine kinds as shown in left figure.

Hana: I remember the same thing in Peg Solitaire British.

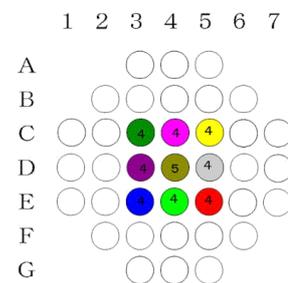
Taro: The same color stones are collected into nine color holes at the center as shown in right figure.



Hana: Said conversions are used.

Taro: Then, the result is shown in right figure.

Hana: Arabic numerals show the number of collected stones.



Taro: It is so.

Hana: The hole with even number stones is the hole with no stone, and

the hole with odd number is the hole with one stone.

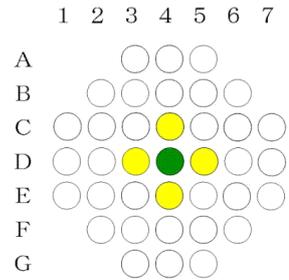
Taro: Then, only one Green stone D4 will remain as shown in left figure.

Hana: It is not the same as Peg Solitaire British.

Taro: Therefore, the game cannot be ended when Green stone D4 is First stone.

Hana: Because the game cannot be ended by Virtual rule, the game should not be able to be ended by Usual rule.

Taro: Do you think which stone is First stone in Usual rule?



Hana: I presume that Stone D5 is First stone.

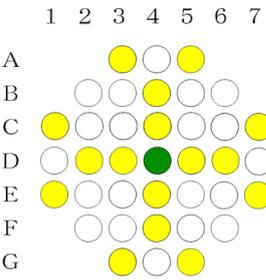
Taro: Strictly speaking, other correct answers exist.

Hana: Because C4, D3 and E4 is symmetrical to D5, stones C4, D3 and E4 too are able to become First stone.

Taro: Besides, it is still.

Hana: The other?

Taro: I think that equivalent stones to stones C4, D3, E4 and D5 are able to become First stone, too.



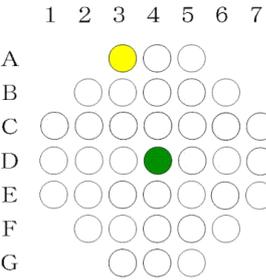
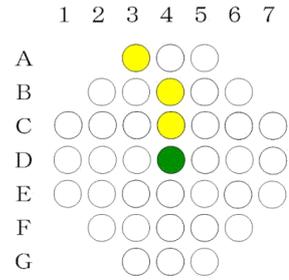
Hana: It doesn't necessarily become so by Usual rule though it is certainly so by Virtual rule.

Taro: The game has the possibility to be ended by Usual rule if it is possible by Virtual rule.

Hana: So.

Taro: Only possibility.

Hana: Then, there will be a possibility that the game is ended when either of



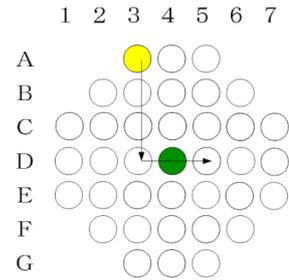
Yellow stones in left figure is First stone.

Taro: Then, let's examine Last stones based on above First stones.

Hana: There are a lot of stones to be examined.

Taro: Considering symmetry, it suffices if only three Yellow stones

in right figure are examined.

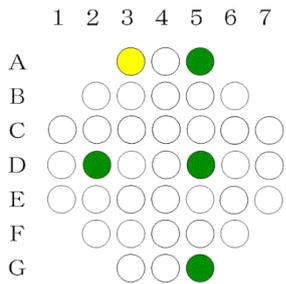


Hana: Then first of all, when Yellow stone A3 is First stone, which is Last stone?

Taro: Yellow stone A3 is transferred to equivalent D3 and is moved to D5 by removing Green stone D4.

Hana: Green stone D5 becomes Last stone when Yellow stone A3 is First

stone.



Taro: Green stones equivalent to Green stone D5 shown in left figure may become Last stone, too.

Hana: Can those four Green stones all become the last stones even by a usual rule?

Taro: The limitation of Peg Solitaire French is looser than Peg Solitaire Diamond as explained later.

Hana: What do you mean by it?

Taro: It is presumed to be possible also in Peg Solitaire French because the combination of First stone A3 and Last stone A5 or D2 was possible in Peg Solitaire Diamond.

Hana: Why can you declare that?

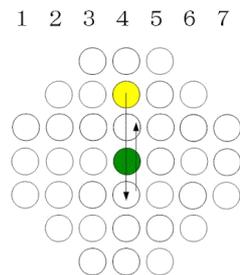
Taro: It is a mere expectation because it is not well-grounded.

Hana: The combination of First stone A3 and Green stone G5 or D5 was impossible in Solitaire 41.

Taro: Therefore, do not you think only the combination should be examined?

Taro: By making the examination of the possibility back, next let's think which is Last stone when Stone B4 is First stone.

Hana: Which is Last stone?

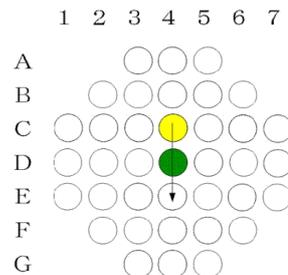


Taro: Stone B4 is transferred to equivalent hole E4 and is moved to hole C4 by removing Stone D4 .

Hana: Stone C4 is Last stone.

Taro: As shown in left figure, Green Stones C1, C7 and F4 equivalent to Stone C4 are Last stone, too.

Hana: Indeed, either of four Green stones in left figure is Last stone when Stone B4 is First stone.

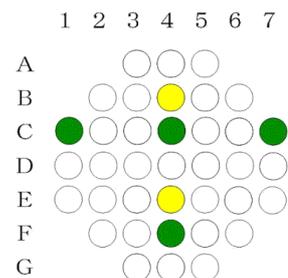


Taro: Finally, let's think which stone is Last stone when Yellow Stone C4 shown in left figure is First stone.

Hana: Only Green stone E4 remains when Stone C4 is moved to E4 by removing Green stone D4.

Taro: Therefore, either of Green stone E4 and its equivalent stones is Last stone, too.

Hana: This looks like Last stones when Stone C4 is First stone.



Taro: You noticed the merit very much.

Hana: If it is rotated by 180 degrees, these Last stones are completely corresponding to Last stones when Stone C4 is First stone.

Taro: If Hole C4 is rotated by 180 degrees, it becomes Hole E4.

Hana: Stone B4 is the same as Stone C4 in the point called First stone.

Taro: If right figure is rotated to the left by 90 degrees, Green stones in right figure is completely corresponding to Last stones in left figure when Yellow Stone A3 is First stone.

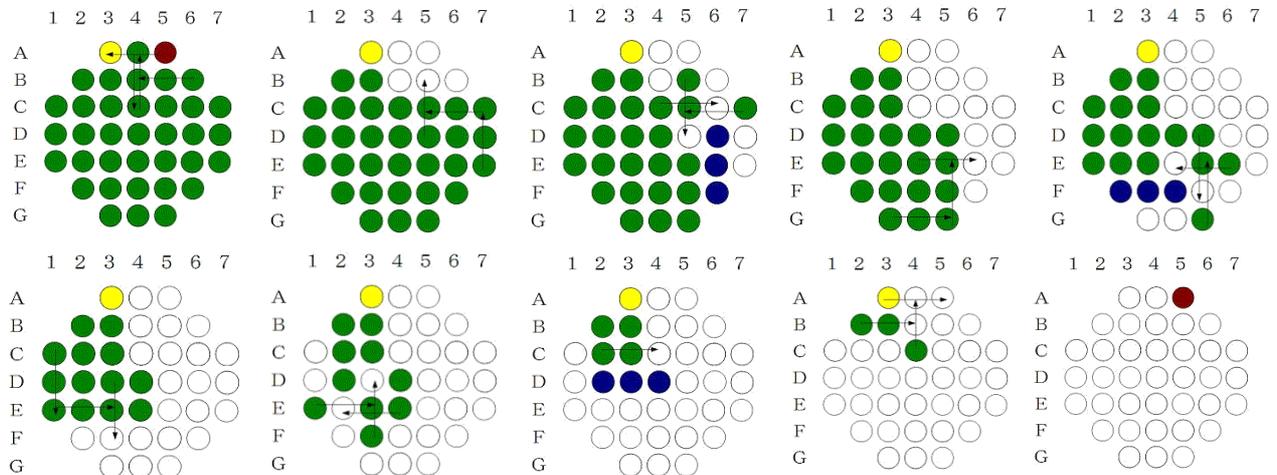
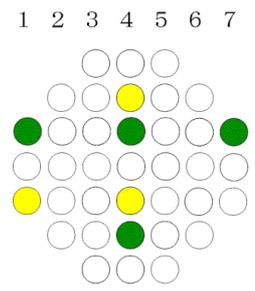
Hana: What do you mean by it?
 Taro: In right figure, if either of Yellow stones B4, E1 and E4 is First stone, either of Green stones is Last stone.

Hana: The reason is that First stones B4, E1 and E4 are mutually equivalent.

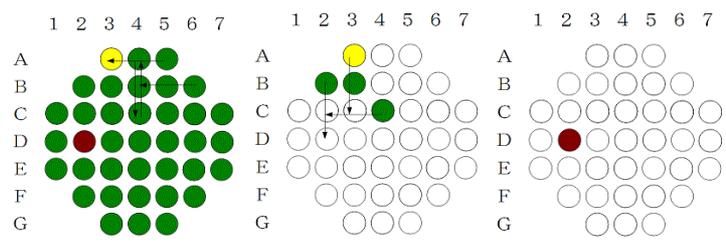
Taro: It is so. It is automatically derived from equivalence and symmetry.

Hana: By the way, how do it become by Usual rule?

Taro: Then, the example of First stone A3 and Last stone A5 is first presented as follows.

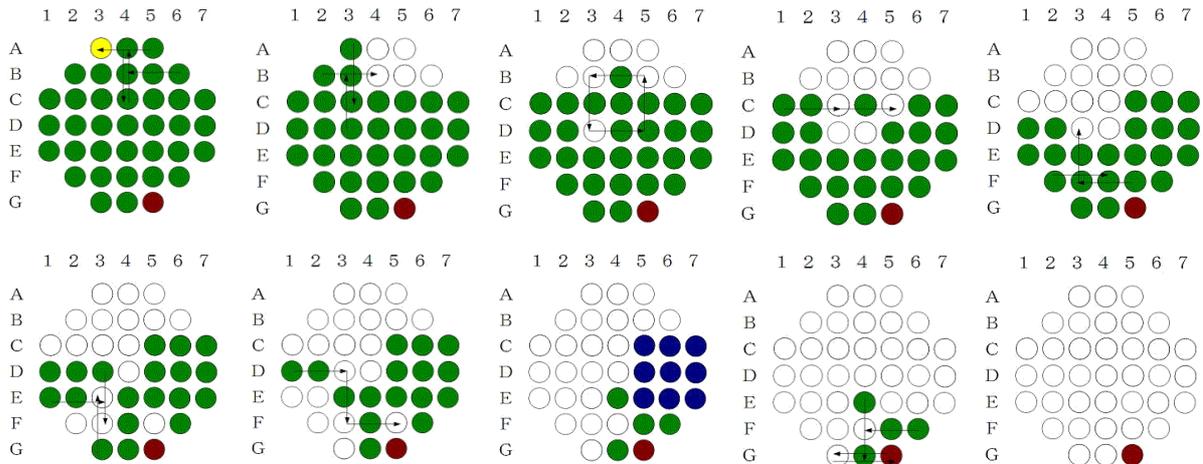


Taro: The example of First stone A3 and Last stone D2 can be facilitated by transforming the example of First stone A3 and Last stone A5 as shown in the figure below.

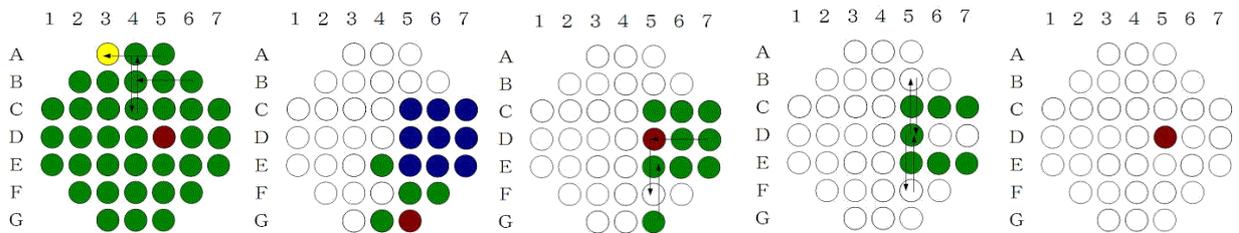


Hana: As for the example of First stone A3 and Last stone D2, only the movement of the last three stones is different from the example of First stone A3 and Last stone A5.

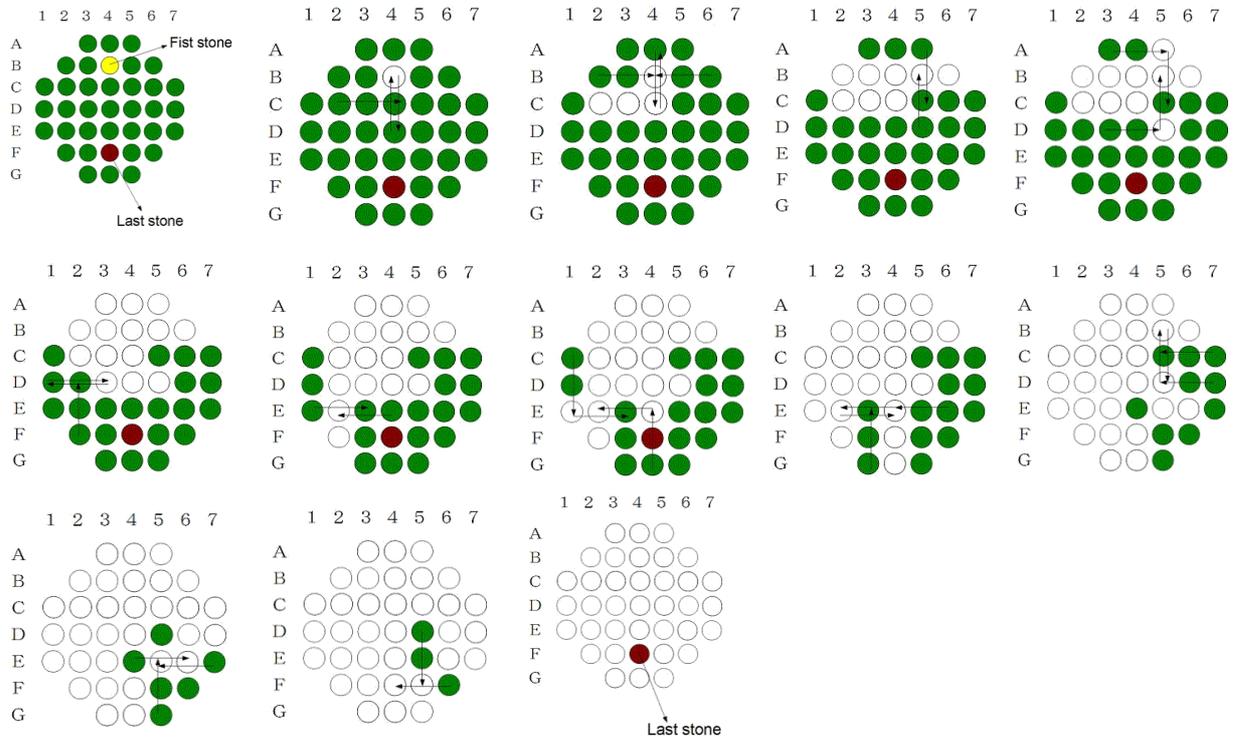
Taro: Next is the example of First stone A3 and Last stone G5.



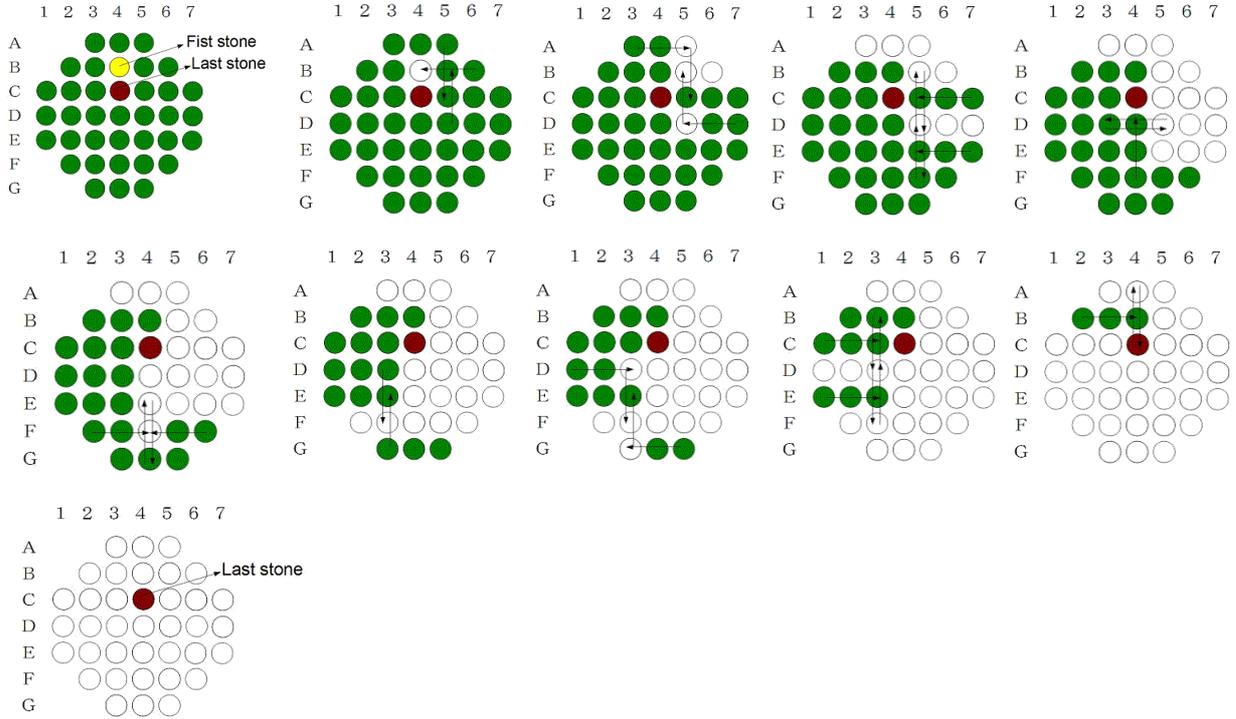
Taro: The example of First stone A3 and Last stone D5 can be facilitated by transforming the example of First stone A3 and Last stone G5 as shown in the figure below.



Taro: The last example is First stone B4 and Last stone F4 as shown in the figure below.



Taro: I had forgotten the example of First stone B4 and Last stone C4 as shown in the figure below.



Taro: I had further forgotten the example of First stone C4 and Last stone E4 as shown in the figure below.

