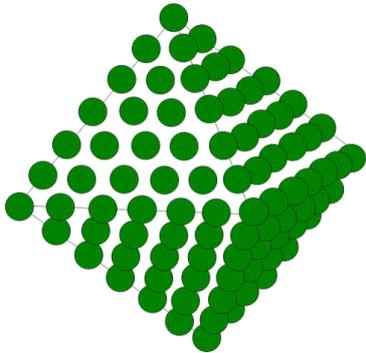# Peg Solitaire Solid Octahedron

Taro: Peg Solitaire Solid Octahedron (in the following, PS Octahedron) is formed by making Peg Solitaire Diamond to solid. In the following, "Peg Solitaire Diamond" is recorded "PS".
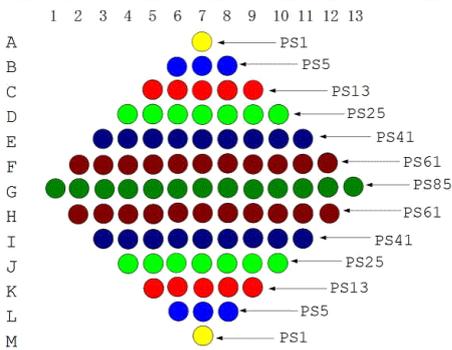
Hana: Please explain plainly.

Taro: PS Octahedron is formed by sequentially piling PS1, PS5, PS13, PS25, PS41, PS61, PS85, PS61, PS41, PS25, PS13, PS5 and PS1 as shown in right figure.
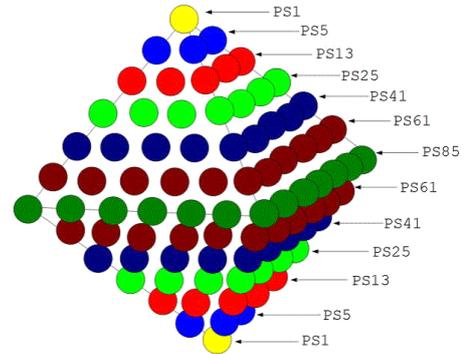
Hana: Isn't this a cube?

Taro: This is a regular octahedron.

Hana: What is the reason though you selected not PS Octahedron 129, but PS Octahedron 377?

Taro: It is because there is a possibility that the solution exists in PS Octahedron 377 though the solution doesn't exist in PS Octahedron 129.

Hana: Please explain the reason more in detail.

Taro: All outside stones in PS1, PS5, PS13 and PS25 can not be moved though all outside stones may be moved in PS41. However, stones necessary to move outside stones remains in PS61, PS85 and PS61.

Hana: In a word, insufficient stones in PS1, PS5, PS13 and PS25 are supplemented with stones that remain in PS61, PS85 and PS61.

Taro: In PS1, PS5, PS13 and PS25, respectively one, three, four and three stones are insufficient. And because PS1, PS5, PS13 and PS25 are piled up twice, 22 stones in total are insufficient. On the other hand, in PS61, PS85 and PS61, respectively five, twelve and five stones, 22 stones in total remain.

Hana: It says very much. The number of insufficient stones and the
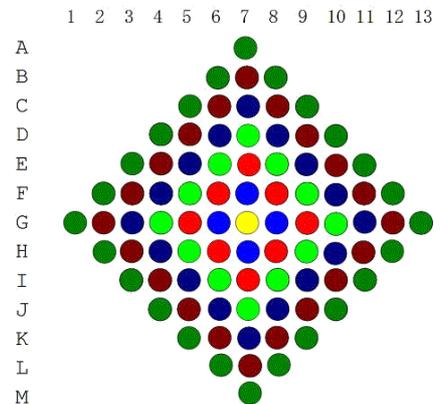
number of remaining stones are corresponding by chance.

Taro: It is so, and I can not express it except saying a chance agreement.

Hana: As for remaining stones, is it actually possible to become insufficient stones ?

Taro: We can forecast the possibility of the existence of solution by applying Virtual rule to this PS Octahedron.

Hana: Please explain to me how to apply Virtual rule to this PS Solid 13.

Taro: In Virtual rule, each stone of PS Octahedron is also equivalent to either of stone of PS Cube 3 (3 × 3 × 3 = 27) as shown in right figure.

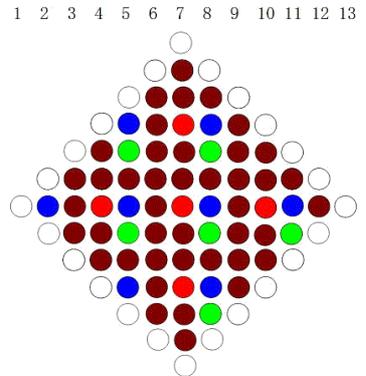These light red, light green and light blue stones are on PS11.

PS11
PS13
PS11
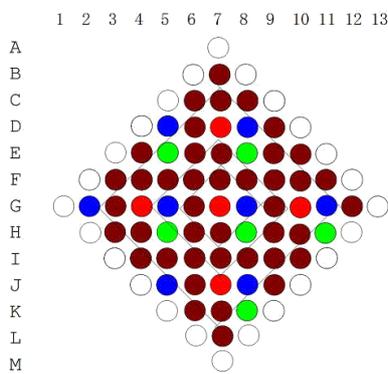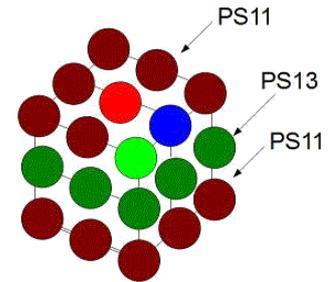
Hana: Please teach me how to transfer all stones of PS Octahedron 377 to PS cube 3.

Taro: Because PS cube 3 is a cube that consists of six square sides, we have to examine only nine stones on one square side.

Hana: Do not it become fewer?

Taro: We have to examine only three colore, namely, light red, light green and light blue stones as shown in left figure, considering the symmetric figure.

Hana: It is very troublesome to total the number of stones of each colors.

Taro: Because only the number of stones of each color on PS5, PS13, PS41 and PS61 is totaled, it is not too difficult.

Hana: Why has PS1, PS25 and PS85 etc been excluded ?

Taro: According to Virtual rule, three colore, namely, light red, light green and light blue stones on PS61 do not exist on PS1, PS5, PS25, PS41, PS85, PS61, PS25, PS13 and PS1 from the left sequentially.

Hana: There are five light red stones, six light green stones and eight

light blue stones on PS61, there are five red stones, four light green stones and five light blue stones on PS41, there are one light red stone, one light green stone and two light blue stones on PS13 and there are one light red stone and one light blue stone on PS5.
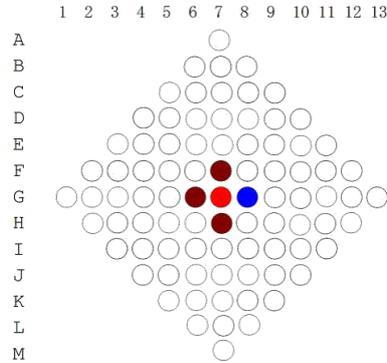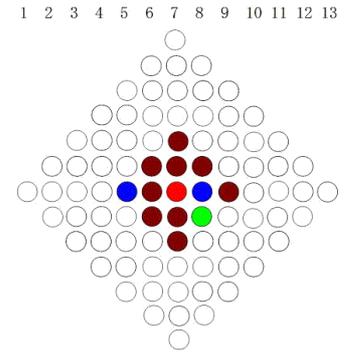
Taro: Then, the number of light red stones is twelve (12) in total, the number of light green stones is eleven (11) in total, and the number of light blue stones is sixteen (16) in total.

Hana: According to Virtual rule, no stone remain in each hole of Cube 3 when the number of stones in the hole is even. Then eight bright green stones remains at the corner of Cube 3 as shown in right figure.
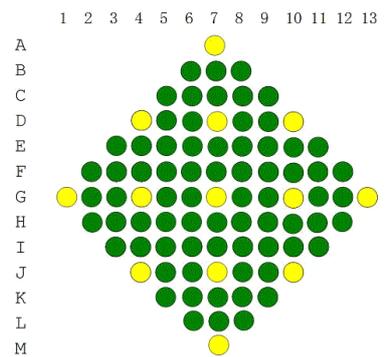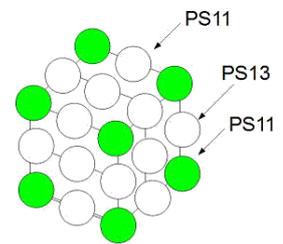
Taro: Do not you forget the stone at the center of Cube 3?

Hana: How is the number of stones at the center of Cube 3 calculated ?

Taro: The number of yellow stones of PS85, PS25 and PS1 is respectively 13, 5 and 1. And, because there are two PS25 and two PS1, the total number of yellow stones is 13+2*5+1*2=25, and an odd number. Then, one yellow stone remains at the center of Cube 3.

Hana: Then, eight bright green stones at the corner of Cube 3 and one yellow stone at the center remain after all.

Taro: Because eight bright green stones at the corner of Cube 3 are converted into one stone at the center according to Virtual rule, two (even number) stones will remain at the center adding up the one yellow stone. When paraphrasing it, no stone remain in Cube 3.

Hana: What does it mean ?

Taro: It means that any stone is acceptable as First stone and it's equivalent stone to the First stone is Last stone in Virtual rule.

Hana: I memorize that First stone was not acceptable any stone in Usual rule.

Taro: Yes it is so. First stone is not limited by Virtual rule as stated above, and limited only by Usual rule.