Relativistic perihelion shift

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It is assumed that an arbitrary position in the gravitational field belongs to the inertia system which has the time axis t' with the light's speed C and the radial axis R' with the relative speed $U(R')$ for the static system. The static system has the time axis t with the light's speed C and the radial axis R. Then, since the world space distance dS^2 is preserved for the coordinate transformation to the inertia system from the static system, the following formula is formed.

C': Light's speed at the arbitrary position in the inertia system (gravitational field)

C: Light's speed in the static system (non-gravitational field)

i: Imaginary ($i^2 = -1$)

$$dS^2 = (iCdt)^2 = dS'^2 = (iCdt')^2 + (U(R)dt')^2$$

$$d\theta = 0, \quad d\Phi = 0$$

Note that the inertia system to which the arbitrary position belongs is different by the position. And also the static system is the non-gravitational field.

Since light's advancing distance is invariant without depending on the selection of the inertia system, the following equation is formed.

$$C'dt' = Cdt$$

Then, the following equation (6) is formed.

(6) $\quad C'^2 = C^2 - U^2 \ , \quad U = U(R)$

On the other hand, when the mass $m \ll M$ with the speed v moves into the gravitational field of the mass M from the infinite remote position, the following equations are formed.

$g = GM/R^2$: Gravitational acceleration

v : The mass speed at the infinite remote location

V' : The mass speed in the gravitational field

F : Gravitational force acting on the mass m

$$(iCdt')^2 + (Udt')^2 + (vdt')^2 = (dS')^2 = (iCdt')^2 + (V'dt')^2$$
$$(Udt')^2 + (vdt')^2 = (V'dt')^2$$
$$V'^2 = v^2 + U^2$$
$$mV'^2/2 = mv^2/2 + mU^2/2$$
$$d(mv^2/2)/dR = 0$$
$$d(mV'^2/2)/dR = d(mU^2/2)/dR = F = mg = mGM/R^2$$
$$dU^2/dR = 2GM/R^2$$
$$U^2 = 2GM/R$$

Then, the equation (6) is rewritten as the following.

(7) $C'^2 = C^2 - 2GM/R$

When $C'^2 = 0$, $C^2 - 2GM/Rs = 0$, $Rs = 2GM/C^2$

Rs : Schwarzschild radius

Since mass advancing distance is invariant without depending on the selection of the inertia system, the following equation is formed.

$$V'dt' = Vdt$$
$$(dt/dt')^2 = 1 - U^2/C^2$$
$$V'^2 = V^2(dt/dt')^2 = V^2(1 - U^2/C^2)$$

$\gamma' = 1 - e$ $\quad 2GM/V^2R = 1$ $\quad U^2 = 2GM/R'$ $\quad \delta = \theta - \omega$

e: eccentricity

When $\gamma' < 1$, the following expressions hold.

$$V'(d\omega/dt) = -(dU^2/dR)\cos\delta = V'(d\omega/d\theta)(d\theta/dt)$$
$$d\theta/dt = V'\cos\delta/R'$$
$$d\omega/dt = (d\omega/d\theta)(V'^2\cos\delta/R') = 2GM\cos\delta/R'^2$$
$$d\omega/d\theta = 2GM/V'^2R' = 2GM/R'V^2(1 - 2GM/R'C^2)$$
$$R'\cos\delta = \gamma'R \qquad Rs = 2GM/C^2$$
$$= \cos\delta/\gamma'(1 - (\cos\delta/\gamma')(Rs/R))$$

When $Rs/R \ll 1$, the following expressions hold.

$$d\omega/d\theta \cong (\cos\delta/\gamma')(1 + (\cos\delta/\gamma')(Rs/R))$$

$$= \cos\delta/\gamma' + (\cos\delta/\gamma')^2(Rs/R)$$

$$\int d\omega \cong \int_0^{2\pi}(\cos\delta/\gamma' + (\cos\delta/\gamma')^2(Rs/R))d\theta$$

$$= \int_0^{2\pi}(\cos\delta/\gamma')d\theta + \int_0^{2\pi}(\cos\delta/\gamma')^2(Rs/R)d\theta$$

$$(\cos\delta/\gamma')^2 \cong 1$$

$$= \int_0^{2\pi}(\cos\delta/\gamma')d\theta + \int_0^{2\pi}(Rs/R)d\theta$$

$$\int_0^{2\pi}(\cos\delta/\gamma')d\theta \cong 2\pi + 2\delta_{\gamma}e$$

$$\cong 2\pi + 2\delta_{\gamma'}e + 2\pi Rs/R$$

$2\delta_{\gamma'}e$: Elliptic Perihelion shift

$2\pi Rs/R$: Relativistic perihelion shift

When $e \cong 0$ and $Rs/R \ll 1$, the following expressions hold.

$\gamma' = 1 - e \cong 1$, $\delta = \theta - \omega \cong 0$, $\cos\delta/\gamma' \cong 1$

$$d\omega/d\theta \cong 1/(1 - Rs/R) \cong 1 + Rs/R$$

$$\int d\omega \cong \int_0^{2\pi}(1 + Rs/R)d\theta = 2\pi + 2\pi Rs/R$$

$2\pi Rs/R$: Relativistic perihelion shift